Adding Unlike Fractions the Fast Way

In Book 5 we saw how adding *unlike* fractions is much harder to do than adding *like* fractions. We took time to explain why it is harder to do, and showed how to do it.

There is a faster way to add unlike fractions, but we didn't put it into Book 5, because we thought that understanding what we were doing was more important than just getting the answer as fast as possible.

But once we *understand* how adding unlike fractions works, it is OK to use the fast way to do it.

To make things a little simpler here, let's just call the numerator the "top" and the denominator the "bottom."

Here is the fast way: To get the new bottom, just multiply the two old bottoms together. The new top is the sum of the cross-products of each top times the *other* bottom.

Like so many other things in math, the explanation seems confusing at first, but once we have done a few examples, we see how easy it actually is.

We'll start with a simple one:

$$\frac{1}{2} + \frac{1}{3} =$$

The new bottom is the product of the two old bottoms, 2 and 3:

$$2 \times 3 = 6$$

To get the new top, we multiply the top of one times the bottom of the other for both fractions:

$$1 \times 3 = 3 \qquad 1 \times 2 = 2$$

Then add them together:

$$3 + 2 = 5$$

The new top is 5, and the new bottom is 6, so we have:

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

Let's look at a more complex example:

$$\frac{3}{7} + \frac{5}{9} =$$

The new bottom is 7 x $9 = 63$.

The new top is 3 x $9 = 27$ plus 5 x $7 = 35$; and $27 + 35 = 62$

So we have:

$$\frac{3}{7} + \frac{5}{9} = \frac{62}{63}$$

We can use this example to review our previous method for adding unlike fractions. Remember that the first thing we had to do was to find the lowest common multiple of the two denominators. This fast way finds a common multiple, but it's not always the lowest one. In this example, however, it just happens to be the lowest common multiple, 63.

Then we had to find equivalent fractions for $3/7$ and $5/9$. Whatever we had to multiply the bottom of either fraction by to get the common denominator, we had to multiply the top of that fraction by the same number. Since we have to multiply the bottom of the first fraction, 7, by 9 to get 63, we have to multiply the top, 3, by 9 also, to get an equivalent fraction. For the other fraction, we have to multiply the bottom, 9, by 7 to get 63, so we have to multiply the top, 5, by 7 also.

But this is exactly what we did using the fast way!

Let's try an example where multiplying the old bottoms together to get the new bottom does not give us the *lowest* common multiple:

$$\frac{3}{4} + \frac{7}{10} =$$

Here the new bottom is 4 x 10 = 40, and the new top is 3 x 10 = 30 plus 7 x 4 = 28. The new top is 30 plus $28 = 58$. So we have:

$$\frac{3}{4} + \frac{7}{10} = \frac{58}{40}$$

This is the correct answer, but it is not in its lowest terms. We can see that both top and bottom can be divided by 2, and when we do that, we have:

$$\frac{3}{4} + \frac{7}{10} = \frac{29}{20}$$

If we had done this problem by first finding the *lowest* common multiple, 20, the first equivalent fraction would have been $15/20$, and the second one, $14/20$. And when we add these together we get $29/20$, the same result.

We can use the same fast method to *subtract* unlike fractions, also. We just have to be careful about which fraction the negative sign goes with. If we associate the negative sign with the top part of the fraction it is in front of, the process is just like addition. Let's look at an example:

$$\frac{3}{4} - \frac{1}{6} =$$

The new bottom is 4 x 6 = 24; that's the same as when we do addition. The two parts of the new top are 3 x 6 = 18, and -1 x 4 = -4. (We use -1 because the negative sign is in front of the $1/6$.) Now 18 plus -4 = 14.

And our answer is:

$$\frac{3}{4} - \frac{1}{6} = \frac{14}{24}$$

Again we see that our answer can be simplified by dividing top and bottom by two:

$$\frac{3}{4} - \frac{1}{6} = \frac{7}{12}$$

As always, the more practice problems you do, the easier things get.