Learning Math with Kayla

Book 9: Solving long division problems

Vicki Meyer

Illustrator Sue Lynn Cotton

The Learning Math with Kayla Books

Book 1 Adding and subtracting like fractions
Book 2 Multiplying fractions
Book 3 Learning multiplication facts
Book 4 Place values, Multiplying large numbers
Book 5 Adding and subtracting unlike fractions
Book 6 Learning about improper fractions and mixed numbers
Book 7 Dividing fractions
Book 8 Adding and subtracting large numbers
Book 9 Solving long division problems
Book 10 Working with decimals and percents
Book 11 Learning about negative numbers
Book 12 Problem solving!

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About the Kayla Books

The Kayla books tell the story of a fourth grade girl who has gotten so far behind in her math class that she is not able to understand what her teacher is trying to teach her. Her math teacher, Mr. Williams, is aware of how poorly Kayla is doing. He decides a tutor would be the best way to help Kayla learn her math.

In this ninth book, Ms. Gibbs reviews short and long division with Kayla. She also teaches her how to calculate rates (e.g., miles per gallon) using division. Because Kayla studies what she is taught she is becoming more and more math savvy.

There are twelve books in this series. Whether you're a fourth grader, in middle school or in high school; a Mom or Dad or a Grandparent, you can learn along with Kayla.

The story is told by Kayla, right before she goes off to college.

About Kayla

I have been asked if Kayla is a real person. She and others in the books are composites of the many kids I have tutored plus friends. During the time I was writing the Kayla Books, the father of a young man we know was shot and killed. Because of all the sadness I included a section on grieving in this book.

The Author

After Vicki raised six really smart kids, she began studying for her Ph.D. in order to keep up with them. She taught at the university level for 25 years, then began tutoring elementary school students. Vicki soon found a new career for herself, tutoring math for at-risk kids, writing about her experiences, and putting together the Kayla books. You can contact Vicki by going to the website (**www.learningwithkayla.org)** and clicking on "Contact."

Vicki lives with her husband, Ed, in Sarasota, FL.

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And a special thanks to my husband, Ed, for all of his great suggestions, his skillful editing, and especially his patience. I would not be able to complete the books without him.

DEDICATION

To my mother, Phyllis Hurtova, who was prevented from going past the fourth grade due to political unrest in Czechoslovakia, but continued to be a life-long learner.

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Chapter 1

Alexis and Ida

Reader: Alexis is Kayla's Mom and she's telling this part of the story.

It was Monday morning and my day off work. Kayla had already left for school, so now I had some time for myself. I sat down in the big chair in the living room with a book I'd been wanting to read. Just as I opened my book, I heard a knock at the door.

I wondered who that could be. I wasn't expecting anyone. I put my book down, went to the door, and opened it. It was Cleveland's Mom!

"Oh, hello, Miss ... uh…" Oh, no! I knew she was Cleveland's Mom but I couldn't think of her name. I just said, "Please come in."

“Ida,” Cleveland’s Mom said as she walked through the door.

“Oh, of course, Ida, I’m sorry I couldn’t remember your name, but I know who you are. You’re Cleveland’s Mom. I have some coffee ready, would you like a cup?”

“That would be great,” she answered and then said, “Sorry, but I don’t remember your name either. I just know you’re Kayla’s Mom.”

“Alexis,” I answered with a big smile. I’m glad I’m not the only one who has trouble with names. I led the way into the kitchen. As I took a cup from the cabinet, I asked Ida to please sit down. I filled her cup with hot coffee.

I still had a little coffee in my cup, but it was cold. I poured it down the drain and poured a fresh cup for myself. I sat down at the table, wondering why Ida was visiting me. I didn’t want to ask her so I just waited.

“You’re probably wondering why I’m here,” Ida said. “Well I’ve been wanting to thank you for your great daughter, Kayla, for some time now. I thought I’d stop by this morning on the way to work to do just that.”

I didn’t know what to say. I know Kayla is great but why is she thanking me?

“Let me explain what I mean. Why, up until just a few months ago, Cleveland was doing poorly in math. When I tried to help him, he would just say he didn’t need to know math. Then he would tell me he was going to be a basketball player and didn’t I know, basketball players don’t need to know much math,” Ida explained with a frown.

I started to think about Kayla. She wasn't doing well in math either but ever since she was given a tutor she is doing much better. Now she actually seems to like math. But...but I didn't know she was helping Cleveland learn math.

Ida continued to tell me about Cleveland. "Why, just in the last few months, I noticed Cleveland is doing his math homework and his math grade has improved. When I talked with him about it, he said it was because of Kayla.

"Cleveland told me that Kayla knows a lot of math tricks and she is showing them to him. Why he even has a multiplication grid that Kayla must have given him. It's already almost half filled out. Can you imagine!"

I didn't know quite what to say. I thought of saying that Kayla is helping me with math too, but I didn't want to say that. Instead I said, "Cleveland is teaching Kayla how to play basketball and I'm grateful for that. You see, her father would have taught her but he's…well, he's not here."

I don't know why I said something about Willie. Oh no, I'm beginning to cry. I thought I was all done crying but…

Ida put her hand on my arm and asked me why I was crying. I told her - I told her everything! It all poured out. I told her about Willie, and I even told her about me not being able to take care of Kayla when she was just a newborn because Willie died and my own momma died too - right after Willie died.

I even told her about my sister, Marie, who had come all the way from Colorado to take care of me and little Kayla too because I couldn't take care of her. I don't know why I told her all those things, but I did. And after I said all that, I felt a little better, but…but I was still crying. I couldn't help it.

What must Ida think of me crying like this? I tried to pull myself together and think of something to say but all I could think of was Willie and I began to cry even more.

Ida put her hand on my shoulder and said, "I can see you're still grieving."

"Grieving?" I asked. "But…but all this happened so long ago." I answered. "It was almost 10 years ago – Why Kayla will be ten soon!"

I was trying real hard to think of something that would help me stop crying. Then, I thought of the pizza party. Oh good! I thanked Ida for inviting Kayla to her house. I told her that Kayla said she had a very nice time.

Ida replied, "Oh, we enjoyed having Kayla over to our house. She's such a delightful girl. I'm so glad she and Cleveland are becoming friends. She's been such a good influence on him."

I felt a little better but now I was a little embarrassed. Ida was being so nice. There was a lull in the conversation. I was trying to think of something to say to fill it. Then I thought of the pictures Ida had hanging on the wall of her house.

I said, “Kayla told me how nice your house was, especially with all the family pictures hanging on the wall in the living room.”

“Yes,” she said. “As you may know, both my husband and I had children before we married each other. We wanted to make sure all the kids are treated the same. That’s probably why there are so many pictures on the walls - they’re all there!”

We were both quiet for a bit but it wasn’t an awkward kind of silence. Then Ida asked if I had a picture of Kayla’s dad somewhere.

“I do,” I said, “but I keep his picture in the drawer in my room. I keep other pictures there too. And then I started to cry again, this time for no reason at all. I quickly apologized to Ida and said, “I usually don’t go around crying like this.”

“Maybe it’s because you’re still grieving,” Ida said.

Why is she saying that I’m still grieving? It happened so long ago.

And then Ida said, “You see, I’m a social worker and I help people with things like this. Maybe I can help you.”

“Help me?” I asked myself. “How can she help me?” I wondered. I said to myself, “I just have to get over all this. It happened so long ago.”

“I have an idea,” Ida said. “I have some time this morning. Would it be OK with you if I came back in a just a little while?” she asked.

I nodded my head thinking that maybe in a little while, I’ll be all done with this crying and we can have a proper visit. Maybe I’ll make some sandwiches.

Chapter 2

Ida returns

I heard a knock at the door again. That must be Ida. It was almost lunch time. I'm glad I made those sandwiches.

I opened the door and there was Ida with a big black plastic bag, and a smaller bag too. They were filled with - I didn't know with what.

Right away I said, "Please come in," and I moved aside so she could easily come in, wondering all the time what she had in those bags.

I didn’t have to wait long to find out. Ida said, “I’m bringing you some picture frames and wall hangers. I thought you might want to hang some of your pictures on your wall. I’ve had a lot of practice hanging pictures. I can help you.”

“Hang my pictures up? But…but I keep them in my drawer. In that way, I look at them just when I’m ready to look at them.”

I heard myself say, “just when I’m ready.” Well, what if Kayla is “ready?” I never thought of that! She should be able to look at the picture of her...of her grandmother if she wants to, too.

Right after I said that, I changed my mind a bit.

“Well…maybe Kayla would like to have that picture of her grandmother so she can look at it whenever she’s ready to look at it, too,” I said. “That’s the one picture I’d like to hang up.” And then I quickly added, “But just that *one* picture! I’ll go and get that one picture.”

“OK”, Ida said softly and then she added, “I’ll get a frame and a wall hanger; you tell me where to hang the picture.”

I returned with the picture of my mom, and Ida found a nice frame just the right size. I took some time deciding where to hang the picture. “Right here, right on this wall,” I said as I pointed to a spot on that wall.

Ida hung the picture of Kayla's grandmother right on that wall. We both stepped back and looked at the picture. I said, "Yes, it looks nice there."

Then we both sat back on the couch. Ida began to tell me about a woman who lived a while ago. Her name was Elizabeth Kubler-Ross. She did a lot of work on how people deal with dying and grieving. "I studied about her when I was in school," Ida said. Then she took a card from her purse and handed it to me.

“Here is something she wrote that helped me when my own mother died. It may help you with your losses too,” Ida said. She handed me a card with some words written on it. This is what the card said:

> You will not ‘get over’ the loss of a loved one.
> You will learn to live with it.
> You will heal and you will rebuild yourself around the loss you have suffered.
> You will be whole again, but you will never be the same.
> Nor should you be the same, nor would you want to.”
>
> --Elizabeth Kübler-Ross & David Kessler

After I read it I was quiet. It was as if this woman wrote this for me and…and I don’t even know her and she doesn’t know me! She said I will not ‘get over’ the loss of Willie. That’s for sure; how could I get over the loss of Willie - and my Momma too - but I was thinking of Willie right now. Why, every time I look at Kayla, I think of Willie. You see, she has his eyes.

I didn’t say anything to Ida. I just kept looking at those words. I reread them again, but this time, a little slower. I noticed “get over” is in quotes. I read further.

She says I’ll be whole again but I’ll never be the same. That’s for sure. She said “I wouldn’t want to be and I shouldn’t be. That’s right! How could I ever be the same? I’m raising our daughter without him, and she is missing out on having a father. I didn’t want it to be this way. Willie would have loved Kayla so much. I continued reading:

“You will learn to live with it. You will heal and you will rebuild yourself around the loss you have suffered.” I don’t know if I healed but I did rebuild myself around the loss. I may not have done such a good job with that though. This woman, who I don’t even know, wrote this for me. I kept reading.

“You will be whole again, but you will never be the same. Nor should you be the same, nor would you want to.” Well, I don’t feel whole. But yes, she’s right. I don’t want to be the same but - but how do I want to be?

Ida was still sitting with me on the couch. I finished reading for the second time what was on the card. But I was still lost in my own thoughts and she was just sitting there, quietly.

Finally, I said to Ida, “I think Kayla would like her father’s picture on the wall too.” And then I added firmly, “And so would I. She and I can look at him whenever we want.” I went to get Willie’s picture from my drawer and held it for a bit.

Then I brought it to Ida to hang up. She didn't need to find a frame for it. It was already in a nice frame.

Why, I remember when Willie gave me his picture. He had already bought a real nice frame for it. He said he was smiling when the picture was taken because he was thinking of me.

Ida hung Willie's picture right near the picture of my Mother but not real close. Then I went back to get the rest of the pictures: the one of Auntie Maria, the picture of me when I was a little girl, and the picture of Kayla when she was just a baby and the picture of Kayla right now.

Right in the space between my mother and Willie, Ida hung the picture of Kayla smiling, I didn't tell her where to hang it but yes, that's where she belongs. After the pictures were hung, we went into the kitchen and ate our sandwiches.

For some reason, I felt a lot better, I talked about Willie, telling Ida about all the fun things we did. Why, I even laughed a little. When we were finished eating, we went back into the living room. All of my pictures were hanging on the wall. We stood back and looked at them all.

Reader, Kayla doesn't yet know that her Mom and Cleveland's Mom hung up all her family pictures. That happened after she had already left for school. Now back to the story; Kayla is telling it this time.

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Chapter 3

My multiplication grid

When I entered the tutor room, I could see Ms. Gibbs had my multiplication grid on the table. After I greeted her, I put my five times facts into the grid. They weren't hard. I remember when I first studied them, I would multiply the number by ten and then take half of that and that was what five times that number is.

That works for real big numbers too – Ten times eight hundred is eight thousand, half of that is four thousand so five times eight hundred must be four thousand. Get it?

I didn't put four thousand in my grid, though. At that time I was just writing in the numbers that fit into my grid and my grid just goes up to multiples of eleven. Eleven times five is fifty-five and that's the biggest number I wrote in my grid on that day.

Ms. Gibbs didn't put the grid back right away so I was able to count the empty boxes. I was surprised! There weren't many of them at all. Why, I really should be able to finish my grid before I finish fourth grade!

Multiplication Facts

	1	2	3	4	5	6	7	8	9	10	11
1	1	2	3	4	5	6	7	8	9	10	11
2	2	4	6	8	10	12	14	16	18	20	22
3	3	6	9	12	15				27	30	33
4	4	8	12	16	20	24	28	32	36	40	44
5	5	10	15	20	25	30	35	40	45	50	55
6	6	12		24	30	36	42		54	60	66
7	7	14		28	35	42	49		63	70	77
8	8	16		32	40			64	72	80	88
9	9	18	27	36	45	54	63	72	81	90	99
10	10	20	30	40	50	60	70	80	90	100	110
11	11	22	33	44	55	66	77	88	99	110	121

*Reader, here you can see how many empty boxes are left in Kayla's grid. And how many empty boxes are in your grid? I hope you're filling in **your** grid!*

"Would you like to learn your three times tables for next time?" Ms. Gibbs asked. Before I answered, I counted all the boxes I still needed to fill in. There were ten boxes, but I only had to learn five new facts - and I know most of them already.

I thought a little before I answered, "Yes, Ms. Gibbs, but … but I was wondering if I could learn more than one number this time, maybe I can learn my six times tables too. Six is twice what three times is so I'll just need to double what three times is. It won't be too hard." Then I told her I want to make sure I filled in my grid before I finish fourth grade.

Reader, however old you are, learn your times tables. If you're still in school, you'll use them in middle school, in high school and in college – and when you go shopping too. And if you're no longer in school, you'll use them for sure when you shop. They're not so hard to learn.

"Kayla, are you sure you want to learn more than one number for next week?" Ms. Gibbs asked. I nodded my head. I was sure - well I was almost sure.

Ms. Gibbs said I could, so now I'll be studying the three times and the six times tables too. Next week, I'll be able to put more numbers in my grid.

These are the boxes that are still blank for the **3**'s and **6**'s:

3 x 6 = 18
3 x 7 = 21
3 x 8 = 24

6 x 3 = 18
6 x 8 = 48

Why, I'll be almost done!

Hmmm… Maybe, just maybe…

Chapter 4

Division

"Now let's start on our lesson. Today I'm going to review division. Let's start with the terms used."

I frowned. "I don't like terms. I always get them mixed up," I explained.

"Kayla, it's much more important to understand what division does than to know the terms, but...but it's always good to know what the terms mean. In that way, you can talk about numbers using these terms and besides - you just might have them on a test someday." Ms. Gibbs smiled as she said that. "There aren't many - only four. You'll see, it won't be too hard."

Ms. Gibbs explained the terms: "The **dividend** is the number being divided, the **divisor** is the number doing the dividing and the **quotient** is the answer. The answer, the quotient, is the number of times the divisor goes into the dividend. And if the divisor doesn't go into the dividend evenly, there is a **remainder**."

Reader, a good way to remember the terms is to keep using them - that's what Ms. Gibbs does.

"In the first example problem below, the **dividend,** the number being divided, is **10.** The **divisor**, the number doing the dividing, is **2** and the **quotient**, the answer, is **5**. That just means there are **5** twos in ten. The divisor goes into the dividend evenly. There is no **remainder**.

“There are two common ways of writing this:

$$\begin{array}{r} \mathbf{5} \\ \mathbf{2\overline{)10}} \end{array} \qquad \text{or} \qquad \frac{\mathbf{10}}{\mathbf{2}} = \mathbf{5}$$

“Both mean there are **5** twos in ten.

“Let’s try to visualize what division means. I have some pennies we can use here in my folder. They will help you better understand what we’re talking about.

“Here we have ten pennies and we want to know how many groups of two pennies we can make. We can take away two pennies at a time until we run out of pennies, and count the number of times we took two pennies. It would be five, of course. This shows how division is just repeat subtraction,” Ms. Gibbs said.

“We can see that there are two pennies in each of the five piles.

“And you know that if ten divided by two is five, then ten divided by five must be two. If we took five pennies away from ten and we did that twice, we would run out of pennies:

“We can see that there are five pennies in each of the two piles.”

I was looking at those coins and following along with what Ms. Gibbs was saying.

She continued, “Another way of saying this is, ‘If five twos fit into ten, then two fives fit into ten.’

“And division can be thought of as the inverse of multiplication. Two times five is ten and ten divided by five is two.

“Similarly, one hundred divided by four is twenty-five; that means that twenty-five fours fit into one hundred:

$$4\overline{)100}\;=\;25 \qquad \text{or} \qquad \frac{\mathbf{100}}{\mathbf{4}} = \mathbf{25}$$

“And because you know about money, you know that one quarter equals **25** cents and one dollar equals **100** cents. This equation says that one-fourth of one dollar is **25** cents, so there must be four quarters in one dollar. This makes sense, because ‘one quarter’ means the same thing as ‘one-fourth.’ ” Ms. Gibbs took more money from the envelope to show me:

=

Then Ms. Gibbs explained about the remainder. “The remainder is just the number left over if the divisor doesn’t go into the dividend a whole number of times. For example, **3** does not go into **10** a whole number of times. It goes in **3** times with a remainder of **1**. If

we have ten pennies and take away three at a time, we can do this **3** times and then there will be **1** penny left over:

"It makes sense to me," I said, "But I don't know if I can remember all those terms."

I don't think Ms. Gibbs heard me, because she just continued to explain about division:

"When we divide a large number by a divisor with more than one digit in it, there are two ways to find the answer. I'll show you both ways and you can decide which one you like better. The first one is the subtraction method."

DIVISION using the subtraction method

"This way subtracts multiples of the divisor from the dividend until we can't subtract any more. We keep track of the number of divisors we subtract. The answer to the division problem, the quotient, is the total number of divisors that fit into the dividend. If the divisor does not go into the dividend evenly, there will also be a remainder."

Subtract? But I thought we were supposed to *divide*. I wondered, but I didn't ask.

Ms. Gibbs saw the puzzled look on my face. She said, "As always, an example will help to clarify things. Let's divide **512** by **32**:

$$32\overline{)512}$$

"The easiest multiples to use in this way of dividing are multiples of ten. Ten **32**'s equal **320**, and we can subtract that from **512**. We keep track of the number of **32**'s we subtract by writing that number down to the right of the multiple we are subtracting. Here's what the problem looks like after our first subtraction":

```
32)512
    320    10
    192
```

"We have subtracted ten 32's from 512 and have written down the 10 to the right of the 320. After subtracting these ten 32's, we still have 192 left that we have to subtract 32's from before we can't subtract any more.

"We could subtract 32 over and over, but there is a faster way. We ask ourselves, "How many 32's might fit into 192?"

I didn't answer, because I didn't know what to answer, but Ms. Gibbs did:

"Well, five is another multiple that is almost as easy as ten. You know that five times something is just half of ten times something. So five times 32 is half of 320, or 160. That's less than 192, so let's subtract five 32's, or 160, next:

```
32)512
    320    10
    192
    160     5
     32
```

"Because 10 + 5 = 15, we have subtracted fifteen 32's from 512, and we have one 32 left over. We now subtract this one, being sure to keep track of it by putting a 1 under the 10 and the 5. We can see that there is nothing left over after we subtract, and our final problem looks like this:

$$
\begin{array}{rl}
32\overline{)512} & \\
\underline{320} & 10 \\
192 & \\
\underline{160} & 5 \\
32 & \\
\underline{32} & 1 \\
0 &
\end{array}
$$

“Adding up the numbers of **32**’s we subtracted gives us a total of **16**:

$$10 + 5 + 1 = 16$$

“There are sixteen **32**’s in **512.** We can check this by multiplying **16** x **32** to see that this does equal **512.**”

This way of solving a division problem clearly shows us that division really is just repeated subtraction, in the same way that multiplication is just repeated addition.

LONG DIVISION using the algorithm

“The second way of solving this problem is called ‘long division’ and I’ll show you an algorithm for achieving it.”

There’s that word “algorithm” again, I thought. I’m glad I know what it means. It’s just a simple rule that someone made up a long time ago to help kids like me do math – well, it helps everyone do math, doesn’t it?

Ms. Gibbs continued, “There are six steps in this algorithm which are repeated until the division is completed”:

1. **Estimate**
2. **Test Divide**
3. **Multiply**
4. **Subtract**
5. **Compare**
6. **Bring Down**

“The best way of explaining what each step means is by doing an example, so I’ll redo the division problem we just did, but this time using long division. We start in the same place:

$$32\overline{)512}$$

“Starting at the left of the dividend, we **Estimate** the first number that **32** fits into. It does not fit into **5**, but it does fit into **51**. We make an ‘educated guess’ at the number of **32**’s that fit into **51**. This is not hard, because one **32** obviously fits, and we know two **32**’s equal **64**, which is bigger than **51**, so two ***32’s*** do *not* fit. So only one **32** fits into **51**. Our first **Test Divide** gives us a **1.**

“We write the **1** over the **1** of the **51** in the dividend:

$$\begin{array}{r} 1 \\ 32\overline{)512} \end{array}$$

“The next step is **Multiply**. We multiply that **1** times the divisor, **32**, and write the answer below the **52** in the dividend:

$$\begin{array}{r} 1 \\ 32\overline{)512} \\ \underline{32} \end{array}$$

“The next step is **Subtract**. We subtract **32** from **51** and get **19**:

$$\begin{array}{r} 1 \\ 32\overline{)512} \\ \underline{32} \\ 19 \end{array}$$

"Now we **Compare** the result of the subtraction with the divisor. If it is larger than the divisor, that means more than one 32 could have fit into 51, and we would have to erase the 1 in the quotient and replace it with a higher guess.

We don't have to do that here, however. 19 is less than 32 so we can go ahead to the next step.

The final step of the algorithm is **Bring Down**. We bring down the next digit from the dividend, which is a 2, and we put it right next to the 19:

```
     1
32)512
   32↓
   192
```

"Now we repeat the same series of steps, but this time asking how many 32's fit into 192. This again involves an 'educated guess.' We can get a rough idea of how many 32's fit into 192 by ignoring the last digit in both numbers. How many 3's fit into 19? 6 is the answer, but we know this is just a rough estimate, and it could be too large. After all, we ignored the 2 in 32. Let's be careful and choose 5 for our **Estimate** this time. For our **Test Divide** we put the 5 above the 2 in the dividend, and **Multiply** it times the 32 to get 160. We put this below the 192:

```
    15
32)512
   32↓
   192
   160
```

“Now we **Subtract**:

$$\begin{array}{r} 15 \\ 32\overline{)512} \\ \underline{32}\downarrow \\ 192 \\ \underline{160} \\ 32 \end{array}$$

When we **Compare** the **32** we got from subtracting, we see it is *not* smaller than the divisor. That means our guess of **5** was too small. We erase the **5** and replace it with a **6**. We also have to erase the **160** we got when we multiplied by the **5**, of course.

When we multiply **32** by **6**, we get **192**, and now our problem looks like this:

$$\begin{array}{r} 16 \\ 32\overline{)512} \\ \underline{32}\downarrow \\ 192 \\ \underline{192} \\ 0 \end{array}$$

“As you can see, there is no remainder. **32** goes into **512** evenly, so there are exactly sixteen **32**’s in **512**.”

“Ms. Gibbs, I think I like this second way better,” I said, and then I explained why. “That’s the way Mr. Williams taught us. But I just couldn’t understand that guessing part. It didn’t seem right that I was supposed to just guess. And besides, I didn’t know what to guess! So I just started drawing pictures instead.

“But now I get it! Guessing is what I’m *supposed* to do. But not any old guess. Whatever I guess has to use the math I already know to make it an *educated* guess. Why, it’s even part of the algorithm, except it’s called **Estimate** instead of **Guess**!”

Ms. Gibbs smiled and said, "So just use the long division algorithm if you like that better than the subtraction method. That's the one I like better, too.

"Kayla, it's time for you to try a long division problem by yourself."

"OK," I said, "but can I look at the list of steps in the algorithm? I don't think I can remember them all."

"I have the algorithm printed out for you, Kayla. Here are the steps":

1. **Estimate**
2. **Test Divide**
3. **Multiply**
4. **Subtract**
5. **Compare**
6. **Bring Down**

"You can look at this list when you're doing the practice problems. But soon, you won't need it. You'll know them all because you will be using them so much.

"Now here is the long division problem for you to solve**:**

$$23\overline{)6578}$$

I remember the first step. It was **Estimate**. **23** doesn't go into **6**, but it does go into **65**. How many **23**'s fit into **65**? Ms. Gibbs said I could get an educated guess by ignoring the last digit. **2** goes into **6** three times, so I'll guess that **3** fit. I'll find out if this was a good guess when I do the **Compare** step later. For my **Test Divide** I put the **3** above the **5** of the **65**:

$$\begin{array}{r} 3 \\ 23\overline{)6578} \end{array}$$

I didn't have to look at the next step because I just knew I had to **Multiply**.

I multiplied the **3** times the divisor, **23**, and wrote that right under the **65**:

$$\begin{array}{r} 3 \\ 23\overline{)6578} \\ \underline{69} \end{array}$$

"And next comes **Subtract**." Oh-oh! I can't subtract **69** from **65**. I looked up at Ms. Gibbs.

"Kayla, that means your guess was too big. You'll have to erase the **3** and the **69**."

I erased those two numbers and wrote a **2** where the **3** was. Then I multiplied the **2** times the divisor:

$$\begin{array}{r} 2 \\ 23\overline{)6578} \\ \underline{46} \end{array}$$

Now I **Subtract**:

$$\begin{array}{r} 2 \\ 23\overline{)6578} \\ \underline{46} \\ 19 \end{array}$$

"Oh, let's see, I'm supposed to **Compare** the answer to the divisor. It's supposed to be less than the divisor, and it is! My second guess was right!"

"Very good, Kayla," Ms. Gibbs said, "now all you have to do is .."

“Wait, wait, don’t tell me!” I shouted, “I just have to bring down the 7 and then I start again.”

After I brought down the 7, my long division problem looked like this:

```
     2
23)6578
   46↓
   197
```

Ms. Gibbs didn’t say anything but I knew she was watching me. Now I’ll start the five steps over again, but this time I’ll use 197 instead of the 65.

Oh, boy! How many 23’s will fit into 197? Oh wait, I’ll just forget about the last digit of both numbers and guess. So instead of 23 I’ll make that just 2 and I’ll change the 197 into 19.

Now let’s see. I know that 9 x 2 is 18 which is pretty close to 19, so there are about 9 2’s in 19. But last time I did this my guess was too big, so maybe I should guess 8 instead of 9. So I’m going to guess 8 for my **Estimate**. I put the 8 above the 7 in the dividend, like this, for my **Test Divide**:

```
     28
23)6578
   46↓
   197
```

And I remember that **Multiply** and **Subtract** are the next two steps. I multiply 8 times 23, write the answer under 197, and subtract:

```
     28
23)6578
   46↓
   197
   184
    13
```

“Ms. Gibbs, I’m remembering the steps already! Next I **Compare** **13** with **23**, and it’s less, so my guess was right!”

I can see that I just have one more **Bring Down** to do, so when I finish the next series of steps I’ll be all done with this long division problem. I bring down the **8:**

```
     28
23)6578
   46↓|
   197|
   184↓
    138
```

For my last **Test Divide** I have to first **Estimate** how many **23**’s fit into **138.** Like before, I change **23** to **2** and **138** to **13**, and ask how many **2**’s fit into **13**? The answer is **6**, but both times I used this way of getting a rough estimate, I chose the next lower number, and I was right both times. So my **Test Divide** number will be **5** this time. I put a **5** over the **8** in the dividend, and by now I know the next steps are **Multiply**, **Subtract** and **Compare**.

Here's what my problem looked like after I did this:

```
     285
 23)6578
    46↓|
    197|
    184↓
     138
     115
      23
```

Oh-oh! The result of the subtraction is *not* smaller than the divisor! That means my guess of **5** was too small. I should have guessed **6**. I can see already that **6** would fit exactly, because five **23**'s leave exactly one **23** more, and that makes **6**. Here's what my problem should have looked like:

```
     286
 23)6578
    46↓|
    197|
    184↓
     138
     138
       0
```

"My final answer to this long division problem is that **23** goes into **6,578** exactly **286** times," I said proudly to Ms. Gibbs.

"Kayla, I see that you have those five steps of the long division algorithm memorized already!" Ms. Gibbs said.

"Well," I replied, "I had to use them over and over again just to solve that one problem! Now that I know most of my multiplication facts, my guesses are educated ones, aren't they?"

She smiled at me and said, “Yes, they are!”

But now, before we finish for today, I want to show you how we do *short* division.”

SHORT DIVISION

“We use long division whenever the divisor has more than one digit. I’d like to show you a special way to divide by a *single* digit. It’s called ‘short’ division, and it involves much less writing. The algorithm for short division is much simpler, too. But you need to know the multiplication facts for the divisor to do short division.

“If you know the multiplication facts for the divisor, you don’t have to **Estimate**, **Test Divide,** or **Compare**. And you’ll see we don’t have to **Bring Down** either. We can do almost all of the steps of the short division algorithm in our heads!

“Here it is:

1. **Divide**
2. **Multiply**
3. **Subtract**
4. **Insert the Remainder**

“And just as with long division, you repeat these steps until you’ve used all the digits in the dividend.

“The big advantage of short division is that there is so much less writing. The only things you actually write down are the quotient and the remainder for each time through these steps. You **Divide, Multiply** and **Subtract** all in your head.

“Since you know your multiplication facts for **6**, we’ll use that for our divisor. Let’s do this example”:

$$6\overline{)3\ 4\ 9\ 8}$$

I noticed spaces between the numbers in the dividend but I didn’t say anything to Ms. Gibbs about it.

Ms. Gibbs continued, “We know **3** can’t be divided by **6**, but **34** can, so we **Divide 34** by **6**. Because we know our Multiplication Facts for **6**, we know there are **5** sixes in **34**. We put the **5** above the **4** in **34**, then we **Multiply 5** times **6** to get **30**. To get the remainder, we **Subtract 30** from **34** to get **4**, and then **Insert the Remainder**, a smaller version of **4**, to the left and slightly above the **9** in the dividend”:

$$\begin{array}{r} 5\phantom{\ {}^{4}9\ 8} \\ 6\overline{)3\ 4\ {}^{4}9\ 8} \end{array}$$

Oh, now I get it. The spaces leave room for putting in the remainders.

“Now we repeat the process: We **Divide** the **6** into the **49** formed by the remainder and the next number in the dividend. **6** goes into **49** eight times; we put the **8** above the **9** in the dividend. Now we **Multiply 8** times **6** to get **48**; **Subtract 48** from **49** to get the remainder of **1**, and **Insert the Remainder**, **1**, to the left of the **8**:

$$\begin{array}{r} 5\ 8\phantom{{}^{1}8} \\ 6\overline{)34{}^{4}9{}^{1}8} \end{array}$$

“Finally, we **Divide** the **6** into the **18** formed by the remainder and the **8**, and we get a **3** with no remainder. We put that **3** over the **8**

$$\begin{array}{r} 5\ 8\ 3 \\ 6\overline{)34{}^{4}9{}^{1}8} \end{array}$$

in the dividend and we have our answer:

"**6** goes into **3,498** exactly **583** times. We can check that this is correct by multiplying **583** times **6,** because multiplication and division are inverses:

$$\begin{array}{r} {}^{4\,1} \\ 583 \\ \underline{\times 6} \\ 3498 \end{array}$$

"If **3,498** divided by **6** is **583**, then **6** times **583** must be **3,498**, and it is.

"Now, Kayla, it's time for *you* to try a short division problem. You don't have to say the steps of the algorithm as long as you remember them":

$$9\overline{)2\;3\;0\;4}$$

"OK. **9** goes into **23** twice, with a remainder of **5.** I put the **2** over the **3**, and the remainder to the left of the **0**, but smaller:

$$\begin{array}{l} 2 \\ 9\overline{)23^{5}0\;4} \end{array}$$

"Now how many **9**'s go into **50**? Well, **5** x **9** = **45**, so there are five **9**'s and the remainder is **5** again. I put a **5** above the **0** and a small **5** to the left of the **4**:

$$\begin{array}{l} 2\;5 \\ 9\overline{)23^{5}0^{5}4} \end{array}$$

"Finally, **9** goes into **54** exactly **6** times, so here's how my problem looks when it's solved:

$$\begin{array}{r} 2\;5\;6 \\ 9\overline{)23^{5}0^{5}4} \end{array}$$

“And I know my answer is right, because $9 \times 256 = 2{,}304$.

“That’s very good, Kayla,” Ms. Gibbs said, “you are learning new things very quickly, but please remember that if you don’t practice what you just learned, you could forget just as quickly.”

“I won’t forget that,” I told Ms. Gibbs, “I always do my practice problems and the more I do, the easier they get!”

I turned to leave but Ms. Gibbs said, “Kayla, I’m not finished yet. There is one more thing I’d like you to know. It’s important!”

So I stopped and turned right around.

Chapter 5

The word 'per'

"Kayla, I want you to know the importance of the word "per." Remember, when we work with fractions, the word 'of' is a hint to multiply, like one-third *of* twelve means you should *multiply* one-third *times* twelve."

I nodded my head. I remembered Ms. Gibbs's saying that.

Ms. Gibbs continued, "Well, 'per' is another word that gives a hint, this time when we're working with division problems. If you look it up in the dictionary, 'per' means 'for each.' So if we know some amount for several things, but we want to know it 'for each' thing, we have to *divide* the total amount for the several things by the number of things."

I frowned a little because I didn't understand – not really!

Ms. Gibbs must have seen my frown, because she said, "Once again, an example will help you understand this idea better.

"Suppose you were babysitting for one of your neighbor's children, and your neighbor gave you $\$\mathbf{12}$ for $\mathbf{3}$ hours of babysitting. In this example, we know the total amount of dollars, and the several things is 'hours.' To find 'dollars per hour,' or 'dollars for each hour,' we have to *divide* $\$\mathbf{12}$ by $\mathbf{3}$ hours:

$$\frac{\mathbf{12}\text{ dollars}}{\mathbf{3}\text{ hours}} = \mathbf{4}\,\frac{\text{dollars}}{\text{hour}}$$

"The result is called a *rate*. When babysitting you earn money at the *rate* of four dollars per hour."

Hey, that's a lot of money, I thought to myself. I'd be glad to babysit for $\$4$ per hour. Now that I know "per" means "for each," I would get $\$4$ for each hour I would babysit. And if I babysat for 3 hours I would get $\$12$. Wow! And if I babysat for 4 hours…

Ms. Gibbs interrupted my thoughts. "Suppose you wanted to buy a new bicycle that costs $\$120$," she said. "How many hours would you have to babysit to save up this much money?"

Wow, I thought, that would take forever! But then I decided I should be able to figure this out. If I make $\$4$ for each hour, then the number of 4's in 120 is equal to the number of hours I need to babysit! And that's just 120 divided by 4:

$$\frac{120}{4} = 30$$

"I would have to babysit for thirty hours," I said, "That's a lot of babysitting!"

Ms. Gibbs replied, "Well, if you enjoy babysitting, the time goes fast, and eventually you would have a new bicycle that you earned all by yourself.

"Let's look at another example. Suppose two dozen eggs come in a carton with 4 rows of eggs. How many eggs per row in this carton?"

"Well," I said, "I know that one dozen equals twelve, and I can multiply twelve times two to find out how many eggs are in two dozen. I'll label the eggs so I don't get mixed up:

$$\begin{array}{r l} 12 & \text{eggs} \\ \underline{\times 2} & \\ 24 & \text{eggs} \end{array}$$

“So the total number of eggs is $\mathbf{24}$ and the number of rows is $\mathbf{4}$; to find eggs per row, I just divide. I can do that!

$$\frac{\mathbf{24}\ \text{eggs}}{\mathbf{4}\ \text{rows}} = \mathbf{6}\,\frac{\text{eggs}}{\text{row}}$$

Now I’ll see if I’m right. I didn’t ask Ms. Gibbs anything because I think I can figure this out for myself. There are six eggs per row and four rows; four times six equals twenty-four, and that’s the total amount of eggs in the carton.

“There are six rows, with four eggs in each row,” I said, “and there are twenty-four eggs all together.”

“That’s right, Kayla,” Ms. Gibbs said, “and when these cartons are being filled with eggs, they are filled at the rate of six eggs per row.

Ms. Gibbs continued, “Another example of “per” would be: ‘A car gets $\mathbf{35}$ miles per gallon of gasoline,’ which is the same as, ‘A car gets $\mathbf{35}$ miles *for each* gallon of gasoline.’

I nodded my head, and said, “I know that ‘per’ just means ‘for each.’ I know how to do this; I just multiply.”

But Ms. Gibbs gave me a funny look and said, “To calculate ‘miles per gallon’ we *divide* the total number of *miles* the car has driven by the total number of *gallons* of gasoline it used.”

“Oh, I meant to say ‘divide’,” I said.

“Yes, you divide,” said Ms. Gibbs. Think of the logic in this. Suppose the car drove $\mathbf{525}$ miles and used up $\mathbf{15}$ gallons of gasoline. The number of miles per gallon is given by:

$$\frac{\mathbf{525}\ \text{miles}}{\mathbf{15}\ \text{gallons}} = \mathbf{35}\ \frac{\text{miles}}{\text{gallon}}$$

"You know that **35** can be written as **35/1**, so it makes sense that **525** miles on **15** gallons is the same as **35** miles *for each* **1** gallon. The car used up gasoline at the rate of **35** miles per gallon. Do you see that, Kayla?"

I nodded my head. Yes, I can see that – well, I can *sort of* see it. But my head feels full of math right now. I'd rather be playing basketball! Oh-oh, I can hear Ms. Gibbs explaining some more math stuff so I better pay attention.

"Now, just suppose a woman, let's call her Maria, went to the gas station to fill up her car. Before she put any gasoline in her car, Maria saw that the odometer read **56,173** miles."

"What's an odometer?" I asked, because I never heard of that.

"An odometer is an instrument in all cars that keeps track of the total number of miles the car has been driven. Some people write down their odometer readings each time they buy gasoline, but most people don't."

"Oh." That's all I said but I was wondering if I would keep track of the number of miles my car went – if I had a car. My momma doesn't have a car either, but if she did, would she?

"Now, let me continue," Ms. Gibbs said. "Maria took out her notebook and wrote what the odometer said. You see, she's one of those people who always keeps track of the number of miles they drive and the number of gallons of gasoline when they fill up.

“She noticed that the last time she bought gasoline, the odometer read **55,923** miles. After she filled the car this time, she saw that it took **10** gallons of gasoline. How many miles did her car get per gallon since her last fill-up? Do you think you can do this, Kayla?”

I shook my head. I didn’t know and I wasn’t trying to think of it. But I *was* thinking. I was thinking this is a really long lesson…

Ms. Gibbs said, “Kayla, please pay attention. We only have a little more to cover.” And then she continued to explain: “The last time Maria filled her car up, the odometer read **55,923**. Now it reads **56,173**, and after she filled the tank, she saw that it took **10** gallons of gasoline. How many miles per gallon did her car get since the last fill-up?”

Oh-oh! I better make this math stuff my priority. Let’s see, I can do this! I want to find “miles per gallon,” and that means I have to divide how many miles the car went by the number of gallons of gasoline it used up.

I can find the number of miles it went – that’s the difference between the two odometer readings in between trips to the gas station. That’s a subtraction problem:

$$\begin{array}{r} {\scriptstyle 5\;1} \phantom{,173 \text{ miles}} \\ \mathbf{5\not{6},173} \text{ miles} \\ \underline{\mathbf{-55,923} \text{ miles}} \\ \mathbf{250} \text{ miles} \end{array}$$

Now I just have to divide **250** miles by the number of gallons of gasoline used, which is **10**.

I started to divide **250** by **10** like this:

$$10\overline{)250}$$

But Ms. Gibbs stopped me before I got started! She said, "Kayla, remember when we studied multiplying multiples of ten? It was easy when you learned how to do it by counting zeroes."

"Oh, yes!" I said, "I remember multiplying really big numbers that had zeroes at the end."

Reader, if you want to review multiplying by multiples of ten, that's in Book 3.

"Well, dividing by multiples of ten is just as easy," Ms. Gibbs said. "We'll have more to say about this when we study decimals, but for now, just remember that numbers with a zero at the end can be divided by ten just by removing the zero - and numbers with *two* zeroes at the end can be divided by one hundred by removing the *two* zeroes."

"Let's see if this makes sense to me," I said. "When we *multiplied* by multiples of ten we *added* zeroes at the end, and now when we *divide* by multiples of ten we *subtract* zeroes at the end. That must be because multiplication and division are inverses, and addition and subtraction are inverses, too." Then I thought to myself, hey, maybe I *am* getting more math savvy!

"Why Kayla, you are truly getting more math savvy all the time!" Ms. Gibbs said with a smile.

When Ms. Gibbs said that, I had to smile, too, because – well, because that's what I was just thinking! I'm going to forget about basketball for now and just think about math. I'm making learning this stuff my priority – that's because I'm getting sooo math savvy!

Let's see. I can divide **250** miles by **10** gallons by just crossing off the zero in **250**! The car got twenty-five miles *per* gallon," I answered proudly.

"That's right, Kayla. And the number of miles a car drives on one gallon of gasoline is known as the car's 'mileage.'

"Now I'd like you to do the same exercise again - by yourself - and with a different car and with different numbers.

"Another woman, let's call her Alma, went to the gas station to fill up her car. And just as Maria did, she looked at her odometer. It read **97,456** miles. She wrote it in her notebook where she keeps track of all the miles she drives.

"She noticed the last time she filled her car up, the odometer read **96,956** miles. Alma's car took **10** gallons of gasoline, the same as Maria's car. How many miles did her car get per gallon since the last fill-up?"

Like last time, I subtracted the two odometer readings like this:

$$
\begin{array}{r}
{\scriptstyle 6\ 1}\phantom{,456 \text{ miles}} \\
\mathbf{9\not{7},456} \text{ miles} \\
\underline{\mathbf{-96,956} \text{ miles}} \\
\mathbf{500} \text{ miles}
\end{array}
$$

And, just like last time, the car used **10** gallons of gasoline to drive this far. So again, I'll divide **500** by **10** by removing one zero, to get **50** miles per gallon.

Wait! That can't be right; Alma's car went way farther than Maria's and they both used the same amount of gasoline. I don't get this – maybe I did something wrong. I better ask Ms. Gibbs.

But before I had a chance to ask her, she told me to figure out how much the women would have to pay for gasoline. Ms. Gibbs said, “Suppose gasoline costs $**3** per gallon. How much would **10** gallons cost?”

Well, I thought, $**3** per gallon means $**3** *for each* gallon, so if we use up **10** gallons, that’s just $**3** times **10**:

$$\frac{\$\mathbf{3}}{\cancel{\text{gallon}}} \times \mathbf{10}\ \cancel{\text{gallons}} = \mathbf{\$30}$$

Let’s see, the gallons cancel each other out, so the answer is in dollars; each woman would have to pay $**30**.

“But wait! Alma’s car went **500** miles, and Maria’s went only **250** miles,” I exclaimed to Ms. Gibbs.

“That’s right,” Ms. Gibbs replied, and then said, “Why don’t you do another ‘per’ problem: How much does it cost to drive each of these cars one mile? Another way of saying this is, what is the ‘cost per mile’ for each car?”

Okay, I thought, “cost *per* mile” means cost divided by miles. So I said, “Maria’s car cost $**30** to go **250** miles:

$$\frac{\$\mathbf{30}}{\mathbf{250}\mathit{miles}} = \frac{\$\mathbf{3}}{\mathbf{25}\mathit{miles}} = \frac{\$\mathbf{3/25}}{\mathit{mile}}$$

“That works out to three twenty-fifths of a dollar per mile,” I said. “I don’t really know how much that is,” I told Ms. Gibbs.

Ms. Gibbs replied, “Yes, that sounds like a strange amount of money, Kayla. Let’s write the $**3** as **300** pennies, and you’ll be able to do the division problem to find out how many pennies it costs to drive one mile.”

So I substituted $300¢$ for $\$3$, which I can do because that's what $300¢$ is; it's $\$3$. So the cost for gasoline is $300¢$ to drive 25 miles, and I get the cost per mile by dividing:

$$\begin{array}{r} 12 \\ 25\overline{)300} \\ \underline{25} \\ 50 \\ \underline{50} \\ 0 \end{array}$$

"Maria's car costs $12¢$ per mile to drive," I said.

"Very good, Kayla! Now how much does Alma pay per mile in her car?"

Let's see, Alma's car went 500 miles and her cost was the same: $\$30$. So her cost per mile was:

$$\frac{\$30}{500 miles} = \frac{\$3}{50 miles} = \frac{\$3/50}{mile}$$

Like before, three-fiftieths of a dollar is a strange-sounding amount of money, so again I'll change dollars to pennies. It costs Alma $300¢$ to drive 50 miles, so we just have to divide $300¢$ by 50 miles. But wait! We can write the numbers, 300 divided by 50, as a fraction and simplify it:

$$\frac{300}{50}\frac{¢}{\text{miles}} = \frac{30}{5}\frac{¢}{\text{miles}} = 6\frac{¢}{\text{miles}}$$

"This comes out to only $6¢$ per mile. That's only *half* as much as Maria's car!"

Ms. Gibbs said, "Yes, that's right!"

Now's my chance to ask her, I thought. "Ms. Gibbs, how can that be? At first I thought I did something wrong. How can two cars have such different mileages?"

Ms. Gibbs simply said, "Alma's car is a hybrid."

"A hybrid?" I asked, "What's a hybrid?"

Reader, You and Kayla will learn what a hybrid is

on the **Something extra** *page.*

Reviewing what I learned

Today I learned two ways to do long division. The first way I learned is the subtraction method. That way really helped me to see that division is just repeat subtraction. I knew that, well I sort of did, but now I can really see that division is just repeat subtraction!

But I didn't pick that way to do long division. I chose to learn the - well, the usual way to divide – the way that Mr. Williams taught us - and that's the one I'm going to review right now.

Ms. Gibbs has a neat algorithm. It's not something you need to try to memorize though. I'll write it out for you and once you use it a couple of times, you'll know it! You'll know it because - well because it makes sense, how else would you divide?

So first I'll give you the algorithm and then we'll work a division problem and then - you'll probably know the algorithm without ever memorizing it. So here's the algorithm:

1. **Estimate**
2. **Test Divide**
3. **Multiply**
4. **Subtract**
5. **Compare**
6. **Bring down**

First you **Estimate** how many divisors go into the dividend. Remember, an estimate is just an educated guess. Then you test it. This is called a **Test Divide** because it may not be right. Then you **Multiply**, **Subtract**, and **Compare**. If the result of the subtraction is not smaller than the divisor the **Estimate** was too low and you have to repeat **Test Divide**. Once the **Test Divide** passes, you **Bring down** the next number in the dividend.

You just repeat the whole algorithm until there is nothing more to **Bring down**. Then you're done with the problem.

After rereading what I just wrote, division sounds confusing. Did it sound confusing to you, too? If you work along with me, you'll see that it will be much easier than it sounds. That's what Ms. Gibbs always says.

And remember, don't try to memorize the names of the parts of the division problem. It's no fun to memorize. Just say the names and that's how you learn them! That's what I did and now I know them all.

Reader, If you think you know how to do this problem, then just do it and check your answer with mine. If you're not sure, then follow along with me and then next time do the same problem by yourself. If you still can't do it the second time, then do it again and by then, I bet you'll know how to divide.

Here's the division problem:

$$25\overline{)7635}$$

Now I'll work the problem.

Let's see: The first step is to **Estimate** how many times **25** goes into **76** so I can do a **Test divide**. I estimate **3** and put it right over the **6:**

$$\begin{array}{r} 3 \\ 25\overline{)7635} \end{array}$$

The next step is to **Multiply**, **3** x **25**. I already know the answer is **75** because, you see, I know about money, and **75¢** is three quarters.

$$\begin{array}{r} 3 \\ 25\overline{)7635} \\ \underline{75} \end{array}$$

The next step is to **Subtract**. So just subtract the **75** from the **76**. That's easy. It's just **1.**

```
    3
25)7635
   75
    1
```

And then **Compare**. The one is less than the divisor; that's the way it's supposed to be.

Now I just have to **Bring down** the next number, so bring down the **3**. Make sure you put an arrow carefully under the **3**, so you don't get mixed up. So now I have **13**. I need to start over with **Estimate**.

```
    3
25)7635
   75↓
    13
```

That's easy; there are zero **25**'s in **13**, so for **Test divide** I'll just have to put a "**0**" over the **3** in the quotient place. I **Multiply** **0** x **25** = **0** and I **Subtract** that from **13**. I don't really have to **Compare** this time; do you see why? My answer so far is **30** - but wait, I'm not done.

```
    30
25)7635
   75↓
    13
    00
    13
```

There's another number left in the dividend. I have to **Bring down** that last number, it's a **5**, and I make sure I put a straight arrow under it to keep it in the correct column.

```
    30
25)763 5
   75↓ |
    13 |
    00 ↓
    135
```

Let's see, I'll have to **Estimate** how many **25**'s there are in **135** so I can start over with another **Test divide**. I know there are four **25**'s in one hundred and one **25** in **35** so that my educated guess is **5**. So, I'll put the **5** down in the quotient, right above the **5** I brought down.

```
    305
25)763 5
   75↓ |
    13 |
    00 ↓
    135
```

Note: an estimate is just an educated guess. That means you use the math you know to guess – oh, I mean to *estimate*. I know that there are **4** quarters in **1** dollar and **1** quarter in **35** cents so that's how I figured out my estimate of **5** for the number of times **25** goes into **135**.

```
     305
 25)7635
    75
     13
     00
     135
     125
      10
```

Then I **Multiply** again. That **5** that I estimated times the divisor, which is **25**, gives me **125.**

And then I **Subtract.** I have a remainder of **10**.

When I **Compare** the 10 with the divisor, 25, I see that it is less, and that's what it is supposed to be.

And since there are no more numbers in the dividend to **Bring down**, you're done dividing. I'll just have to write **R10** after the answer and then I'll really be all done. Oh, the "**R**" stands for "remainder." So the quotient is this: **305 R10.**

For those of you who did this problem by yourselves, is that what you got? If not, you probably got it wrong because I probably got it right. Wait, I'll just prove my answer is right!

I'll multiply the quotient times the divisor, and I should get the dividend. Let's see if I'm right… I got **7625**.

```
  305
  x25
 1525
 6100
 7625
```

Wait! That's not right but it should be. I did everything right, I know I did! I looked at my division problem and…oh-oh, I figured out what I needed to do.

Reader, did you figure out what Kayla needed to do?

I forgot to add the remainder. So I added it and guess what?

I got it right! $7625 + 10 = 7635$. See? I knew I was right and I *was* right. I just forgot about the remainder! Did you forget too?

Oh, there is one more thing you're supposed to learn. Actually, there is more than one more thing but that's what Ms. Gibbs said. I bet you're getting tired of all this math. I know I was, but please pay attention just for a little bit more. Or if you want, just go get a drink of water and then come back, or if you know how to play a little Sudoku, you can do that too. That's what Ms. Gibbs does when she has trouble concentrating. When you're all done doing those things, then pay attention!

Now I want to show you about *short* division. You can do division the short division way if you have only one digit in the divisor *and* you know the multiplication facts for that divisor. So make sure you know your multiplication facts.

Here is the short division algorithm:

1. **Divide**
2. **Multiply**
3. **Subtract**
4. **Insert the Remainder**

Short division is really fast. That's because if you know your multiplication facts for the divisor, **Divide** is not a guess…you *know* what goes into the quotient. And that means you don't have to do the **Compare** step to find out if your guess was right! The remainder is *always* less than the divisor. And you do the **Multiply** and **Subtract** steps in your head.

Suppose you want to divide the same number, 7635, but the divisor has only one digit: 5, not 25. Remember, you have to know your multiplication facts for 5! Do you?

And when you write down the dividend, please put some space between the digits like I did here:

$$5\overline{)7\ 6\ 3\ 5}$$

Reader, again if you know how to do it, just do it, but check your answer with mine. As before, if you're not sure how to do it, just work along with me. The steps are much easier for short division.

First **Divide:** **5** goes into **7** one time. I put the **1** in the quotient and **Multiply** it times the divisor, **5**. I **Subtract** the **5** from **7** and see that the remainder is **2**. To **Insert the Remainder** I'll put a small **2** right before the **6**, like this:

$$\begin{array}{l} \quad 1 \\ 5\overline{)7^{2}6\ 3\ 5} \end{array}$$

Then I ask myself how many **5**'s are there in **26**? The answer is **5**. So, I put the **5** in the quotient above the **6**:

$$\begin{array}{l} \quad 1\ \ 5 \\ 5\overline{)7^{2}6\ 3\ 5} \end{array}$$

Then in my head I **Multiply** **5** x **5** = **25** and I **Subtract** that from the **26** to get a remainder of **1**, and I **Insert the Remainder** right before the **3**, like this:

$$\begin{array}{l} \quad 1\ \ 5 \\ 5\overline{)7^{2}6\ {}^{1}3\ 5} \end{array}$$

Then I just repeat the steps:
How many **5**'s are there in **13**? That's easy: **2.** So I put a **2** in the quotient right above the **3**:

$$\begin{array}{l} \quad 1\ \ 5\ \ 2 \\ 5\overline{)7^{2}\ 6^{1}3\ 5} \end{array}$$

In my head I **Multiply** **2** x **5** = **10**, and when I **Subtract** that from **13** there's a remainder of **3**. I make it little and **Insert the Remainder** in front of the **5**, so now that's **35**. Get it?

$$\begin{array}{l} \quad 1\ \ 5\ \ 2 \\ 5\overline{)7^{2}\ 6^{1}3^{3}5} \end{array}$$

Then I need to ask myself, how many **5**’s are there in **35**? If you know your multiplication facts for **5** like I do, you know the answer is **7** and it goes in evenly; there’s no remainder. I put the **7** right above the **5** in the dividend, and my answer to this short division problem is **1527**.

$$\begin{array}{r} 1\ \ 5\ \ 2\ \ 7 \\ 5\overline{)7^{2}\ 6^{1}3\ {}^{3}5} \end{array}$$

Like before, we can prove this answer is correct by multiplying the quotient, **1527**, by the divisor, **5**. If we get the dividend, we know we got the correct answer…and we do:

$$\begin{array}{r} {}^{2\ 1\ 3}\quad \\ 1527 \\ \underline{\times 5} \\ 7635 \end{array}$$

Did you get all that? If you did, then you can work short division. If you didn’t, then work it again with me - and again and again if you need to. Whatever you need to do, just learn it because it’s really neat, don’t you think?

Now your head is probably full of math and you probably want to take a rest. I know I did - but Ms. Gibbs had one more thing to show me. I didn’t want to learn that one more thing, but I’m glad I did and so will you be, I bet! So, take a little rest if you really need it. Maybe you can get a drink of water or maybe take a little walk. Then come back and learn about “per.”

The word “per” just means “for each.” So if you hear that if someone can ride a bike **5** miles *per* hour, that means she can ride **5** miles *for each* hour she’s riding a bike. Get it? So what if she rides a bike for **2** hours, how far will she go?

If you answered ten miles, then you got it right. If you answered something else, then you got it wrong.

Here's how I did it and I did it right because Ms. Gibbs said I did. I just thought that if she rides **5** miles in one hour, she has to go twice as far in two hours, and that's **10** miles.

Just remember that "per" means "for each," that's all it means. So when I see the word "per", I just think "for each" and then I can figure out the problem.

For example, if a car gets **30** miles per gallon of gasoline, I just think, OK, the car goes **30** miles *for each* gallon. So, if the car goes **30** miles for each gallon, you just need to multiply **30** times the number of gallons you bought and you'll get how many miles you can go on that number of gallons. If you buy **10** gallons you can go:

$$\frac{\mathbf{30}\text{ miles}}{\mathbf{1}\ \cancel{\text{gallon}}} \times \mathbf{10}\ \cancel{\text{gallons}} = \mathbf{300}\text{ miles}$$

The gallons in the numerator divided by the gallon in the denominator just equals **1**, so the gallons disappear and only the miles are left, and there are **30** x **10** of them and that equals **300**.

So, if someone writes down the number of miles on the odometer every time she buys gas she can easily get the number of miles she drives between fill-ups. Got it? You don't have a car, but just pretend. Say your odometer read **3200** miles right after the last time you filled up your car, and now, after you fill it up again, it reads **3500**. Just subtract **3200** from **3500** and you get **300**. That's how many miles you went between fill-ups.

Now if the car takes **10** gallons to fill it back up, the car went **300** miles on **10** gallons. How far a car goes on one gallon is called the car's "mileage" and it is in "miles per gallon." That word, "per," tells us to divide the first, miles, by the second, gallons:

$$\text{Mileage} = \frac{\mathbf{300}\ \text{miles}}{\mathbf{10}\ \text{gallons}} = \mathbf{30}\ \frac{\text{miles}}{\text{gallon}}$$

And if you want to learn how much money it takes to drive your car, that's another division problem. You can find out how much money it takes to drive one mile; that would be in dollars *per* mile. That is, it's how many dollars it costs *for each* mile you drive Say if the gas costs **3** dollars per gallon and you get **30** miles per gallon, that comes out to **3** dollars per **30** miles:

$$\frac{\mathbf{3}\ \text{dollars}}{\mathbf{30}\ \text{miles}} = \frac{\mathbf{1}}{\mathbf{10}}\ \frac{\text{dollars}}{\text{mile}}$$

But you know one-tenth of a dollar is just ten cents, so now you know that it costs ten cents to drive this car one mile.

Now that you've learned all that, you'll need a lot of practice so you'll remember everything – so PRACTICE!

Practice problems

Some of these problems have a remainder; others don't. Oh, and please don't use your calculator for now. There are not very many problems. Once you're sure you know how to work them, then it's OK to use your calculator. That's what I did.

Short Division:

1. $2\overline{)8\ 7\ 7\ 1}$ 2. $3\overline{)3\ 7\ 9}$ 3. $9\overline{)9\ 7\ 5\ 5}$

4. $5\overline{)9\ 8\ 6\ 5}$ 5. $10\overline{)9\ 9\ 6\ 5\ 0}$

Long Division:

6. $15\overline{)5\ 6\ 7\ 8}$ 7. $37\overline{)4\ 9\ 8\ 5}$

8. $31\overline{)8215}$

9. $57\overline{)66251}$

10. After a very good year, the owner of a business decided to give a total of $2000 to his 25 workers to share equally. How much did each worker receive?

_________dollars per worker

11. Zeo has $10 and he wants to spend it all on gasoline for his car. If gasoline costs $3 per gallon, how many gallons can he buy?

______gallons

12. A farmer in Illinois grew 7,740 bushels of corn on his 36-acre farm. What was the overall yield in bushels per acre?

__________bushels per acre

13. A school has 450 students with an annual budget of $5,197,500. What is the annual cost per student at this school?

__________dollars per student

14. a. Tyree babysits for different families in his neighborhood. If he babysits for three hours, he gets twelve dollars. What is his hourly wage?

___________dollars per hour

b. If Tyree wants to save up one hundred dollars, how many hours of babysitting does he have to do?

_____hours

15. The owners of two cars, one red, the other black, went to the gas station to buy some gas. Before filling up her car, the owner of the red car looked at the number she wrote in her notebook before her <u>last</u> fill-up. It read 12,563. Then she wrote down the odometer reading for <u>this</u> fill-up. It was 12,825. The car needed 11 gallons to fill the tank. I have two questions for you:

a. How far did the car go between fill-ups?

_____miles

b. What was the gas mileage since the last fill-up?

_____miles/gallon

The owner of the black car also kept track of her gas mileage. The number in her notebook before the last fill-up was 21,345, and now it read 21,795. Her car took 9 gallons. Again I have two questions for you:

c. How far did the car go between fill-ups?

_____miles

d. What was the gas mileage since the last fill-up?

____miles/gallon

e. Now after you figured all that out, I have a bonus question for you. What color was the hybrid car?

____

16. Please convert these improper fractions into mixed numbers.

a. $\frac{10}{3}$ =

b. $\frac{24}{5}$ =

c. $\frac{39}{6}$ =

d. $\frac{56}{5}$ =

e. $\frac{66}{8}$ =

f. $\frac{93}{7}$ =

17. Adding and subtracting like fractions – so you don't forget. Watch the signs!

a. $\frac{3}{8} + \frac{5}{8}$ =

b. $\frac{9}{21} - \frac{4}{21}$ =

c. $\frac{6}{9} - \frac{1}{9}$ =

d. $\frac{3}{12} + \frac{5}{12}$ =

18. And please don't forget how to add and subtract unlike fractions:

a. $\frac{1}{3} + \frac{3}{6} =$ b. $\frac{9}{12} - \frac{1}{3} =$

c. $\frac{4}{6} + \frac{1}{3} =$ d. $\frac{5}{8} - \frac{1}{4} =$

19. Or how to multiply fractions:

a. $\frac{4}{6} \times \frac{1}{2} =$ b. $\frac{3}{7} \times \frac{4}{6} =$

c. $\frac{5}{8} \times \frac{3}{4} =$ d. $\frac{9}{11} \times \frac{2}{5} =$

20. And what about dividing fractions? You can't forget how to do that!

a. $\frac{1}{3} \div \frac{3}{5} =$ b. $\frac{2}{3} \div \frac{6}{8} =$

c. $\frac{4}{5} \div \frac{5}{8} =$ d. $\frac{3}{7} \div \frac{6}{5} =$

Something Extra

A “hybrid” is something that is a combination of two or more things. In the animal world, for example, a mule is a hybrid because its mother is a horse and its father is a donkey.

Ms. Gibbs told me all about hybrid cars so now I’ll tell you all about them. You see, there are two kinds of motors: electric and gasoline. Electric motors are more efficient than gasoline motors, but they need batteries to make them go. And when a battery runs out of energy it has to be recharged, just like when a car runs out of gas, the tank has to be refilled.

Well, a hybrid car has both kinds of motors in it. The gasoline motor does two jobs: The first thing it does, it makes the car go, and the second thing it does is charge up the battery. The electric motor just makes the car go, but if you have a hybrid car, you never have to go to the charging station to recharge the batteries.

And because electric motors are more efficient than gasoline motors, and for some of the time the car is running on electricity, you can buy less gas and that’s how you save money. And besides, it’s better for the environment. That’s what Ms. Gibbs says and she knows. You see, she has a hybrid car!

Answers

Short Division:

1. $\begin{array}{r} 4\;3\;8\;5\;\text{ R1} \\ 2\overline{)8\;7\;{}^{1}7{}^{1}1} \end{array}$

2. $\begin{array}{r} 1\;2\;6\;\text{R1} \\ 3\overline{)3\;7{}^{1}9} \end{array}$

3. $\begin{array}{r} 1\;0\;8\;3\;\text{R8} \\ 9\overline{)9\;7\;5{}^{3}5} \end{array}$

4. $\begin{array}{r} 1\;9\;7\;3 \\ 5\overline{)9{}^{4}8{}^{3}6{}^{1}5} \end{array}$

5. $10\overline{)9\;9\;6\;5\;0} = \frac{99650}{10} = 9965$

Long Division:

6. $$\begin{array}{r} 378\;\text{R8} \\ 15\overline{)5678} \\ \underline{45} \\ 117 \\ \underline{105} \\ 128 \\ \underline{120} \\ 8 \end{array}$$

7. $$\begin{array}{r} 134\;\text{R27} \\ 37\overline{)4985} \\ \underline{37} \\ 128 \\ \underline{111} \\ 175 \\ \underline{148} \\ 27 \end{array}$$

8.
$$\begin{array}{r} 265 \\ 31\overline{)8215} \\ \underline{62} \\ 201 \\ \underline{186} \\ 155 \\ \underline{155} \\ 0 \end{array}$$

9.
$$\begin{array}{r} 1162\text{ R17} \\ 57\overline{)66251} \\ \underline{57} \\ 92 \\ \underline{57} \\ 355 \\ \underline{342} \\ 131 \\ \underline{114} \\ 17 \end{array}$$

10. $**80** per worker

11. $3\frac{1}{3}$ gallons

12. **215** bushels per acre

13. $**11,550** per student

14. a. 4 dollars per hour b. 25 hours

15. a. **262** miles b. $23\frac{9}{11}$ miles per gallon

c. **450** miles d. **50** miles per gallon

e. Black

16. a. $3\frac{1}{3}$ b. $4\frac{4}{5}$ c. $6\frac{1}{2}$ d. $11\frac{1}{5}$ e. $8\frac{1}{4}$ f. $13\frac{2}{7}$

17. a. **1** b. $\frac{5}{21}$ c. $\frac{5}{9}$ d. $\frac{2}{3}$

18. a. $\frac{5}{6}$ b. $\frac{5}{12}$ c. **1** d. $\frac{3}{8}$

19. a. $\frac{1}{3}$ b. $\frac{2}{7}$ c. $\frac{15}{32}$ d. $\frac{18}{55}$

20. a. $\frac{5}{9}$ b. $\frac{8}{9}$ c. $\frac{32}{25} = 1\frac{7}{25}$ d. $\frac{5}{14}$

About tutoring

It’s always a good idea when tutoring to include something practical like Ms. Gibbs did. She taught Kayla how to figure out gas mileage. Although Kayla is certainly not old enough to buy gas for a car now, she can use the information to begin thinking about it and perhaps share it with others.

Ms. Gibbs also taught Kayla how she can figure out things about the money she could make babysitting. If Kayla knows the hourly rate, she can calculate how much money she would make babysitting for any number of hours. And if she wants to save up a certain amount of money, she can calculate how many hours she would have to babysit.

Many school age children don’t think that math is practical. Show them that it is.


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