Learning Math with Kayla

Book 5: Adding and subtracting unlike fractions

Vicki Meyer

Illustrator Sue Lynn Cotton

The Learning Math with Kayla Books

Book 1 Adding and subtracting like fractions
Book 2 Multiplying fractions
Book 3 Learning multiplication facts
Book 4 Place values, Multiplying large numbers
Book 5 Adding and subtracting unlike fractions
Book 6 Learning about improper fractions and mixed numbers
Book 7 Dividing fractions
Book 8 Adding and subtracting large numbers
Book 9 Solving long division problems
Book 10 Working with decimals and percents
Book 11 Learning about negative numbers
Book 12 Problem solving!

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About the Kayla Books

The Kayla books tell the story of a fourth grade girl who has gotten so far behind in her math class that she is not able to understand what her teacher is trying to teach her. Her math teacher, Mr. Williams, is aware of how poorly Kayla is doing. He decides a tutor would be the best way to help Kayla learn her math.

In this fifth book, Kayla is able to put 18 more numbers into her multiplication grid using the shortcuts Ms. Gibbs, her tutor, shows her. Ms. Gibbs also teaches Kayla about equivalent fractions, and with this new knowledge, she can now add and subtract unlike fractions.

There are twelve books in this series. Whether you're a fourth grader, in middle school or in high school; a Mom or Dad or a Grandparent, you can learn along with Kayla.

The story is told by Kayla, right before she goes off to college.

About Kayla

I have been asked if Kayla is a real person. She and others in the book are composites of the many kids I have tutored plus myself as a kid *and* as an adult. I remember getting mixed up adding and subtracting unlike fractions as do many of the kids I tutor.

I was an adult when I first tried to shoot a basket. When I was a kid, girls didn't play basketball. But even as an adult shooting my first basket, I was amazed at how heavy a basketball was.

The omelets Kayla's mother made, I make most mornings for my husband and myself. They're good and good for you.

About the Author

After Vicki raised six really smart kids, she began studying for her Ph.D. in order to keep up with them. She taught at the university level for 25 years, then began tutoring elementary school students. Vicki soon found a new career for herself, tutoring math for at-risk kids, writing about her experiences, and putting together the Kayla books.

Elizabeth Meyer (with Vicki on back cover), one of her really smart kids, teaches Classics at Phillips Academy in Andover, MA. When Vicki needs help with a word in her Kayla books, Elizabeth is the one she asks. In this book it was "paradox." In Book 4 it was "savvy," in Book 3 it was "daunting," in Book 2 it was "mnemonic," and in Book 1 it was "mortified."

Acknowledgements

Tom Swartz of Black Oyster Press has generously donated his time and energy to the Hurtova Foundation mission.

Thanks to Charles Daniel for the excellent job he does in proofreading the text. His expertise in mathematics and grammar ensures that both kinds of errors are minimized in the Kayla Books.

And a special thanks to my husband, Ed, for all of his great suggestions, his skillful editing, and especially his patience. I would not be able to complete the books without him.

DEDICATION

To my mother, Phyllis Hurtova, who was prevented from going past the fourth grade by political unrest in Czechoslovakia, but continued to be a life-long learner.

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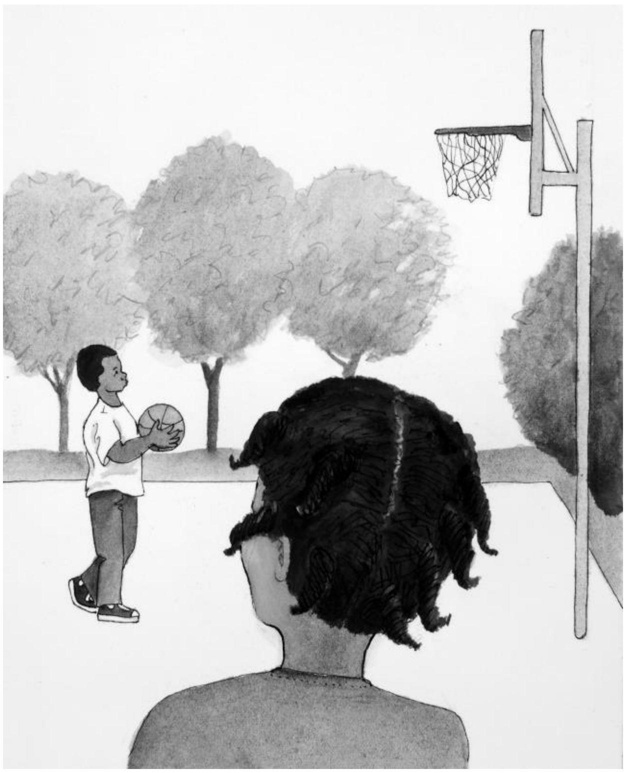
Chapter 1

Basketball

It was early Saturday morning. Momma asked me to go to the store to get some bread for breakfast. “Make sure you get whole wheat bread,” she called out as I was going out the door.

And I did. I know Momma doesn’t like plain old white bread. She says whole wheat bread tastes better and is better for you. I like them both.

On the way back from the store, I saw a boy on the playground playing basketball by himself. “Hey, that kind of looks like Cleveland,” I thought. As I got closer, I saw that it was.

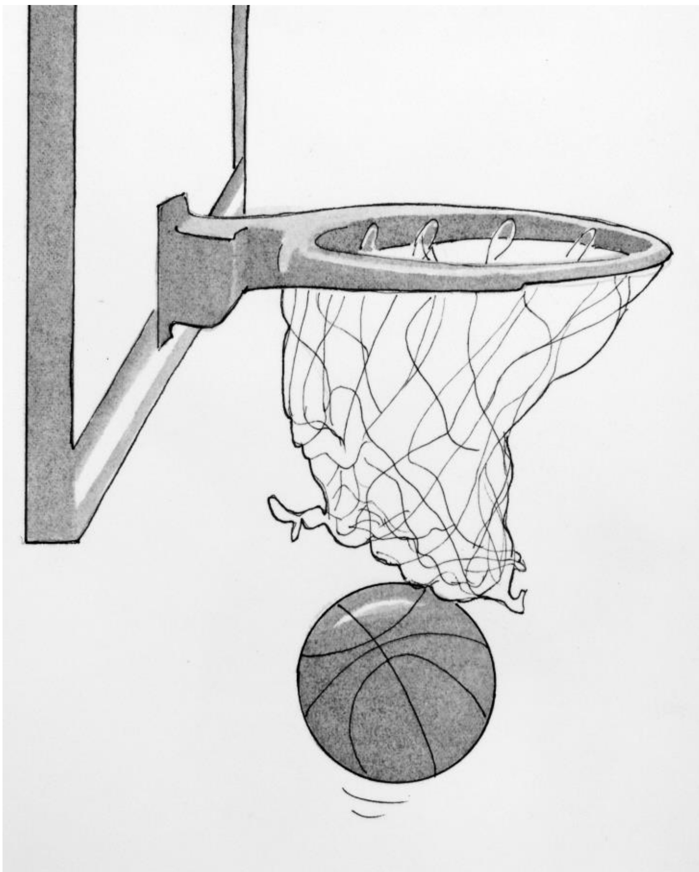


“Hi Cleveland!” I shouted out to him. He looked my way and gave me a half wave. I walked closer so I could watch him play.

I never played basketball before but sometimes I would watch other kids play. The playground was right near my house. I pass it every time I go to the store.

Cleveland was bouncing the ball as he ran around the court. He would try to get the ball in the basket but he kept missing.   That’s because the basket was real high.

But then, while I was watching, he threw the ball right in the basket. It didn’t even hit the side of the basket. It just went right in.



“Wow! How did you do that?” I asked.

“Practice.” That’s all he said. Just that one word.

He made a few more baskets while I was watching but most of the time he didn’t get the ball in. He didn’t seem discouraged though. He just chased after the ball and tried again and again to get it in.

Hmm, maybe if I chased the ball for him, he would let me play with him. I didn’t want to ask him though. So I just put the bread bag down on the ground and started chasing after the ball. If the ball went my way, I caught it and tried to bounce it to Cleveland.

Sometimes I got the ball right to him but most of the times I didn’t. Cleveland didn’t seem to mind though.

After I chased a lot of balls, I felt brave enough to ask. “Hey Cleveland, can I try to get the ball in the basket?”

Cleveland didn’t answer in words. He just bounced the ball my way.   I tried to bounce the ball the way he did but it didn’t go that way. So I just picked up the ball and walked closer to the basket. While I was walking, Cleveland shouted, “Dribble! Dribble!”

Dribble? What does dribble mean? I didn’t want to ask him though. So I got right under the basket, and threw the ball up.   I was surprised at how heavy it was. It didn’t even get close to the basket.

After that, I didn't feel like trying to make another basket. I wanted to stay, though. I tried to return the ball to Cleveland the best I could.

Suddenly the ball was coming right past me. I ran backwards trying to get it. But then I stumbled. I almost fell but I caught myself just in time.

Oh, no! I stepped right on my bag, the bag with the bread in it! And worse still, Cleveland saw me!!

Time for me to go home. I grabbed the bag, and turned to go.

Cleveland called out to me. "Hey, where are you going?"

"Home!" I called back. "I can't play basketball anyway."

“Well if you want to be good at basketball you’ve got to practice, practice, practice,” he shouted as I was leaving.
“That’s what I’m doing.”

As I walked home, I thought of Cleveland. He let me play with him. He didn’t laugh at the way I threw the ball. He didn’t even laugh when I stepped on my bread. And he seemed to want me to stay.

Then I remembered the bread. I opened the paper bag. Oh good, the plastic bag wasn’t ripped. And only about half the bread was squished, the other half was okay. Momma can eat the unsquished part.

Chapter 2

Squished bread

“Kayla, what took you so long?” Momma asked as I walked into the house. “I’m waiting on you for breakfast. Did you get the bread?”

“Yes” I said, “but...”

“But what?” Momma asked.

“It got a little squished but don’t worry, you can eat the good part and I’ll eat the squished part, I don’t mind,” I answered.

“Well you can tell me how the bread got squished while we’re eating our breakfast,” Momma said. “Right now, please set the table and get the butter and jam from the fridge too. I’ll make the omelet.”

Momma makes omelets just the way I like them. First she cracks three eggs in a bowl, then she scrambles them with a little bit of water. Then she pours the eggs into a pan on the stove. Oh, first she puts a little butter in the pan and lets it melt. She puts some cheese and some cooked broccoli on top of the eggs. And then – this is the tricky part – she folds the omelet in half and lets it cook just a little bit more.

After the omelet is all cooked, Momma divides it in half and we eat it! It looks delicious and it tastes even more delicious! Mmmm, having an omelet for breakfast is the best.

But we only have them on Momma’s days off. On the days she goes to work, I eat my breakfast at school.

I put two unsquished pieces of bread on Momma's plate and two squished pieces on mine. I put a little butter and a lot of jam on my bread. It tasted good. I even forgot I was eating squished bread.

While we were eating, I told Momma about my new friend, Cleveland. "Well," I said, "he wasn't really a *brand* new friend. He's my classmate and we had to go to the Time-out Room together and...oops." I stopped talking. I hadn't told my momma about my Time-out. I sorta forgot.

Momma said, "Go on with your squished bread story. I know all about your Time-out. Now tell me about Cleveland."

"Oh!" I exclaimed. Then I just continued on with my story. "Well during our fire drill I taught him a little bit of fractions and - and, then today I watched him play basketball and he let me try to get the ball in the basket. I didn't get it in though. I was chasing the ball for Cleveland and that's how I stepped on the bread."

"Well," Momma said, "Cleveland sounds like a nice boy. Maybe sometime you can invite him to have breakfast at our house."

"Yes, I guess I could. I don't know if he'll come, though." Then I added, "But If he does come, maybe I can teach him some more about fractions."

I didn't tell Momma how nice Cleveland can draw a cat - and an elephant, too. I was thinking about it though.

Hmm. Maybe after he comes over and after we finish our breakfast, he can teach me how to draw that cat and then we can go to the playground and he can teach me how to get the ball in the basket and then maybe...

Momma interrupted my thoughts, “Kayla, please finish eating your breakfast. Your omelet is getting cold.”

Chapter 3

Short-cuts

As I walked into the tutor room, I saw my multiplication grid on the table. Ms. Gibbs said, “Good afternoon, Kayla. How are you?”

“I’m fine, Ms. Gibbs, thank you.”

Then she asked, “Kayla, do you think you’ll be able to put all the nine times tables in your grid today?”

“Oh, yes,” I answered, “I’ve been practicing with the cards you gave me and I learned them all. It wasn’t *so* hard.”

“That’s good. I’ll ask you your nine times tables now. I’m going to skip around a bit and we’ll see how well you do.”

Ms. Gibbs quickly asked me my nine times tables. The first one she asked me was how much four times nine is. I didn’t use my fingers and I didn’t have to write anything down either.

“Four times nine is thirty-six.” While I was answering her, I pictured the answer, “thirty-six,” in my head and quickly added the two digits. I did it real fast. Yep! They added up to nine. I smiled to myself just a little bit.

“And nine times eight?” Ms. Gibbs asked.

“Nine times eight is seventy-two.” Again, I pictured the answer, “seventy-two,” in my head and added the two digits. I knew they would add up to nine but I was just checking.

Then Ms. Gibbs asked me the rest of my nine times tables, one right after the other - real fast. And I answered just as quickly as she asked. It didn't matter if they were in order and it didn't matter which number she said first:

"Nine times nine?"

"Eighty-one."

"And nine times two?"

"Eighteen."

"What about five times nine?"

"Forty-five."

"And nine times six?"

"Fifty-four."

"And nine times seven?"

"Sixty-three."

"And the last one, three times nine?"

"Twenty-seven."

Whew! That was kinda fun, especially since I got them all right. I didn't have time to add the digits up though. But that's OK, I knew they would add up to nine because that's how I practiced them.

I thought Ms. Gibbs would be surprised that I got them all right but she didn't seem to be.   She did say two words, "Very good," but that's all.

I didn't mind though. I got to put twelve more numbers in my multiplication grid.

"Ms. Gibbs, do you have any more short-cuts like that one for the nine times tables?" I asked. "I like short-cuts."

"Well I see you don't have your eleven times tables filled in. The short cut for the eleven times tables is much easier than the one for the nine times tables. Would you like to learn all your eleven times tables?"

"Much easier?" I asked, "Oh yes, I would!"

First Ms. Gibbs explained, "You know that for each successive number you multiply by ten, the answer must be ten more."

I nodded my head.

"Well since eleven is one more than ten, you need to add eleven for *each* of the numbers you multiply by eleven." She emphasized the word, "each."

"See if you can tell me what the short cut is.   I'll start off with this equation," Ms. Gibbs said as she wrote:

$$\mathbf{11 \times 1 = 11}$$

She continued to write the equations as she was saying them.

Eleven times two is eleven more than eleven so that makes twenty-two.

$$11 \text{ x } 2 = 22$$

Eleven times three is eleven more than twenty-two so that makes thirty-three.

$$11 \text{ x } 3 = 33$$

Eleven times four is eleven more than thirty-three so that makes forty ..."

$$11 \text{ x } 4 = 4\_$$

"Four!" I shouted. "I get it!! You just repeat the number you're multiplying eleven by."

Reader, please put the four in the equation for Kayla.

"Yes," Ms. Gibbs said with a smile. "And why do you repeat the number?"

"Uh... Let's see, eleven is one more than ten." For each number you multiply eleven by, you have to make it eleven more so eleven plus eleven is twenty-two. But you don't really have to add eleven to each number - why that would just be repeat addition! If you know the short cut, you can just repeat the digits like this: 22, 33, 44 and so on.

"Oh, and that only works until nine times eleven!" I added. "I'm not sure if that's the best way to explain it but I understand it. Really, I do!"

Ms. Gibbs must have thought so too because then she said, “Yes, that’s how it works, and as you say, just until you get to nine. But you already know how much eleven times ten is, don’t you?”

I do? Oh yes, I do! That’s another short cut. I just add a zero to eleven. Eleven times ten is one hundred and ten!” Then I added, “And that’s because we’re using the base ten system.” Hey! I’m starting to sound real smart.

“Very good,” Ms. Gibbs said, “Now please finish these equations”:

11 x 5 = ___	**11 x 8 = ___**
11 x 6 = ___	**11 x 9 = ___**
11 x 7 = ___	**11 x 10 = ___**

Reader, please finish these equations for Kayla.

“And what about this last one. Do you remember what eleven times eleven is?” she asked.

“Yes! I remember that one,” I answered, “It’s one hundred and twenty-one!”

$$11 \times 11 = 121$$

“I remember that from the day you showed me the multiplication grid for the very first time. You told me there were one hundred and twenty-one boxes. And I remember you said that I was supposed to fill them all in - with numbers!

"And - and I remember thinking at the time I could never be able to fill in one hundred and twenty-one boxes with numbers! Now I think I will be able to do it because - well because, I *am* doing it right now! - aren't I?"

Reader, I hope you're filling out your multiplication grid too, so you can learn along with Kayla. If you haven't begun yet, please get a grid from my website and start ***right away****!*

"Kayla," Ms. Gibbs said, "I think you'll finish your grid before you finish fourth grade. And when you know all your multiplication tables, you'll be able to do other math much faster. Won't that be nice?"

I nodded my head as I finished up all the eleven times tables and put the "121" in the very last box.   Now just because it is the very last box on the grid, didn't mean I was all done. I was getting close, though.

I started to count the boxes I had left but Ms. Gibbs picked up my grid and put it into my folder.

Reader, Kayla didn't have a chance to count the boxes she has left. Can you do this for her? The Multiplication grid is on the next page. How many boxes does Kayla still have to fill in?   ______ *boxes*

Multiplication Facts

	1	2	3	4	5	6	7	8	9	10	11
1	1	2	3	4	5	6	7	8	9	10	11
2	2	4	6	8	10	12	14	16	18	20	22
3	3	6							27	30	33
4	4	8							36	40	44
5	5	10							45	50	55
6	6	12					42		54	60	66
7	7	14				42			63	70	77
8	8	16							72	80	88
9	9	18	27	36	45	54	63	72	81	90	99
10	10	20	30	40	50	60	70	80	90	100	110
11	11	22	33	44	55	66	77	88	99	110	121

"Now I'd like you to learn three more multiplication facts for next week. You can put them on one small piece of paper," Ms. Gibbs said as she handed me a small piece of paper. I wrote the three multiplication facts Ms. Gibbs wanted me to learn on that paper:

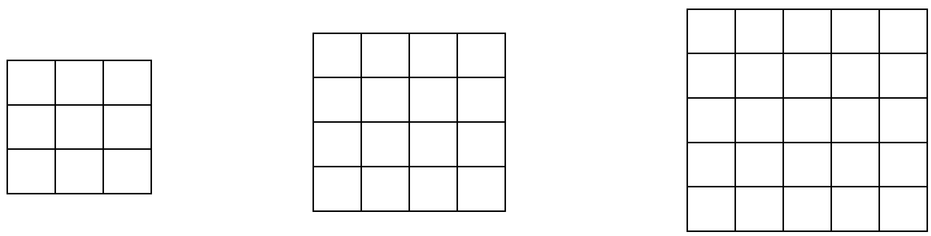
3 x 3 =	9
4 x 4 =	16
5 x 5 =	25

"These numbers can form squares," Ms. Gibbs said.

"Huh? Squares?" I asked.

Ms. Gibbs explained, "Yes, when you multiply any number by itself, the numbers can form a square. Square numbers are written like this: 3^2, 4^2, 5^2 and so on. In fact, another way of saying 'three times three' is just 'three squared'!

"Here's a good picture of these three squared numbers to help you understand what I mean":



3^2 4^2 5^2

“You already know what two squared is. For next week, I want you to learn what three squared, four squared and five squared are.”

I put the paper with the three squared numbers in my pocket and began to leave the room but then I remembered something.

“Oh, Ms. Gibbs, may I please have another multiplication grid? I want to give one to Cleveland.”

“Why, yes, of course. I have a blank one right here,” Ms. Gibbs said as she took the paper from her folder.

I told Ms. Gibbs I wanted to show Cleveland these short-cuts. I think he’s really going to like them. He’ll probably say they’re good tricks but I didn’t tell that part to Ms. Gibbs - I was thinking about it though.

I made sure I said “Thank you,” and I hurried to leave the room.

But then I heard Ms. Gibbs call my name, “Kayla, where are you going?” I turned around looking puzzled.

“It’s still early and we have a lot more work to do,” Ms. Gibbs said. “I want to show you how to add and subtract unlike fractions.”

“Oh”, I said, “I thought it was time to go.” I quietly sat down.

Chapter 4

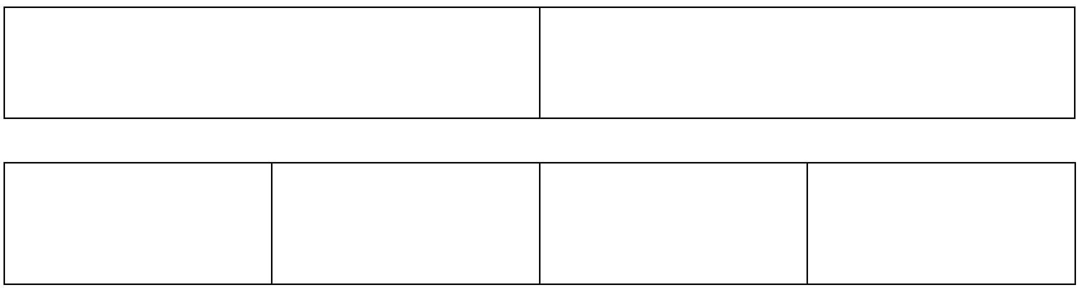
Making equivalent fractions

"Kayla, as you know, the fractions we have been adding and subtracting up till now have had the same denominators. They're called "like fractions." Well, today, I'm going to show you what to do when you have to add and subtract unlike fractions - that is, fractions with *different* denominators," Ms. Gibbs said.

Oh, no! I knew this was coming. I always wondered what I would do if the denominators were different. Now I'm going to find out.

Ms. Gibbs continued, "But first you need to understand about equivalent fractions." She took out some paper from her folder.

She pointed to two fraction bars, one right under the other:



Ms. Gibbs picked up the orange pencil and said "I'll quickly color one section in the first bar. And Kayla, please color two sections in the second bar."

I quickly colored my two sections green.

Reader, please color these sections for Ms. Gibbs and Kayla.

“I want you to look at these two fraction bars. Did you color more than I did?” Ms. Gibbs asked.

“Well it looks like what we colored is the same,” I answered.

“Yes, that’s right, what we colored *is* the same. That’s because these fractions, ‘one-half’ and ‘two-fourths,’ are equivalent fractions. That means they have the same value even though they have different numbers,” she explained as she wrote:

$$\frac{1}{2} = \frac{2}{4}$$

“But…but I don’t get how they could be different and the same at the same time. That doesn’t make sense,” I said.

“Well, yes, it does seem like a bit of a paradox, doesn’t it?” Ms. Gibbs said.

“A pair of what?” I asked.

“A paradox. A paradox is a statement that seems to contradict itself, yet may be true.

“The first syllable is pronounced just like the word “pair” as in a pair of socks. But it’s not a pair of socks, it’s a par-a-dox and it is spelled like this”:

Ms. Gibbs wrote out: “p a r a d o x.”

I looked at what she wrote and then said the word just like Ms. Gibbs said it: “paradox.”

“That’s good,” she said. “You pronounced it correctly. Now maybe this little dashed line I’m drawing in the first fraction bar will make it seem less like a paradox to you:

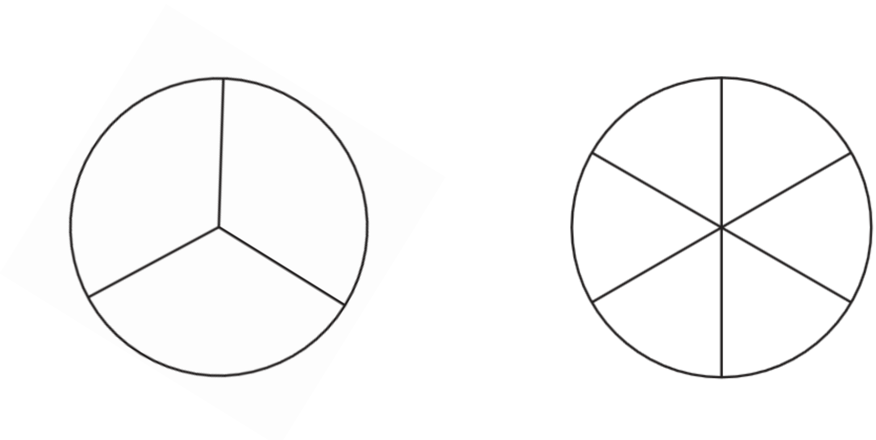
Kayla, now do you see that two-fourths is the same as one-half?”

Reader, can you see it too?

I nodded my head, but said, “But, but…”

I don’t think Ms. Gibbs heard my “Buts.” She just said, “Now let’s continue with what we’re doing. We’ll use pizzas this time.

“Let’s pretend these circles are pizzas. As you can see, they are the very same size. The one on the left is for me and it’s divided into three sections. The one of the right is for you and it’s divided into six sections.   Now let’s pretend I ate two pieces of my pizza and you ate four pieces of yours.”



"They would have to be very small pizzas for me to eat four pieces! I usually just eat two," I said.

"Well, they are very small pizzas," Ms. Gibbs said. Then she continued, "Now since we can't really eat these pizzas, let's just color the pieces we eat brown."

"Brown?" I asked.

"Yes," Ms. Gibbs answered, "Let's pretend brown is the color of the bottom of the pan they're in."

Ms. Gibbs quickly colored two pieces brown, showing the pieces she ate. Then she handed me the brown pencil and said, "Kayla, please use this pencil to show the four pieces you ate."

Reader please use your brown pencil to color two pieces in Ms. Gibbs' pizza and four pieces in Kayla's pizza.

"Now, who ate more pizza?" Ms. Gibbs asked.

"Well, it looks like we ate the same amount, your pieces were just bigger than mine," I answered.

"Yes, as you can see from this and the previous example, fractions can have the same value yet have different numbers. Two-thirds is the same as four-sixths:

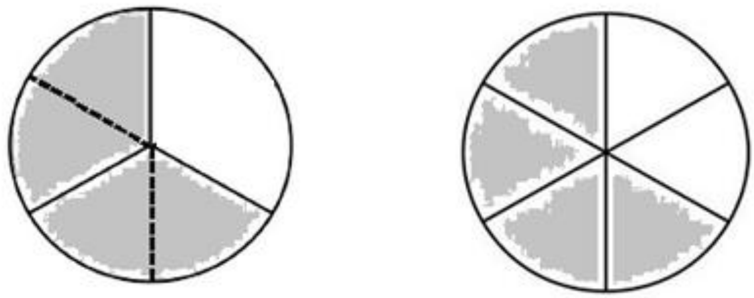
$$\frac{2}{3} = \frac{4}{6}$$

"These pictures are a good way to help you understand how that can be so, now, aren't they?" Ms. Gibbs asked.

“Yes, but…” I kept looking at those pizzas. And I wasn’t just looking at them, I was thinking, too.”

Then I saw what I needed to do!

I reached for my pencil and drew two dashed lines in Ms. Gibbs’ pizza. “There! Now you ate four pieces, too!”



“That’s very good, Kayla. Your dashed lines make it clear that my two pieces were equivalent to your four pieces.”

Reader, the parts of the pizzas that Ms. Gibbs and Kayla ate are shaded in, so you can easily see they ate the same amount, too.

“Now I’ll show you how they’re the same by doing the math. And we’ll use the math you already know. You already know that if the top number and the bottom number of a fraction are the same, the fraction equals one. Isn’t that right?” Ms. Gibbs asked.

I nodded my head.

“And you already know that if you multiply or divide any number by one, the value of the number doesn’t change. Right?” Ms. Gibbs asked.

I nodded my head again. I remember learning that in second or maybe it was third grade.

“With this knowledge, you’ll be able to make fractions have different numbers without changing their values,” she said.

Ms. Gibbs is trying to make it sound so easy. Well this stuff may be easy for her but I bet it won’t be easy for me.

Ms. Gibbs continued, “Now I want you to multiply the fraction “two-thirds” by “two-halves” - which is really one, of course.”

Ms. Gibbs wrote: $\frac{2}{3} \times \frac{2}{2}$

I didn’t pick up my pencil because this was starting to sound too hard. I began to slouch in my chair.

“Two times two is ...?” Ms. Gibbs asked.

“Four,” I answered. I slouched a little more.

“And three times two is ...?”

“Six.” I sat up a little straighter in my chair because I wanted to see what Ms. Gibbs was going to do with the answers I gave her.

She said, “That’s right!” as she finished the equation:

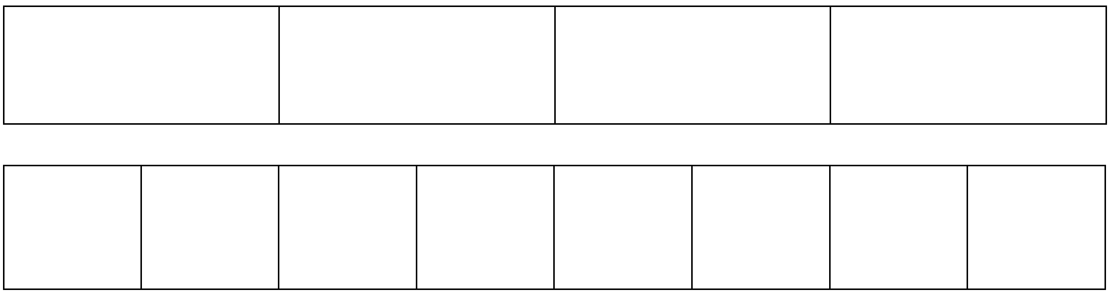
$$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$$

"Two-thirds, the amount of pizza I ate, is the same as four-sixths, the amount of pizza you ate. We changed the numbers in the fraction, two-thirds, to four-sixths, without changing the value - we've made *equivalent fractions*.

Now do you understand how fractions can have the same value but have different numbers, Kayla?"

I wanted to say "Yes" to Ms. Gibbs, but I still wasn't so sure. It still seemed like a trick or something to me.

Ms. Gibbs saw the puzzled look on my face and said "Let's do another problem. Here are two fraction bars, one is divided into four sections.   The other is divided into eight sections.



"Now I'll color in three sections of mine with this red pencil."

Reader, please color three sections of the first fraction bar for Ms. Gibbs.

"Kayla, instead of coloring in your fraction bar right away, why don't you first figure out what fraction of your bar you would *need* to color so that the part you will color is the same as what I colored."

I looked at both fraction bars. One was right under the other. I could see that Ms. Gibbs colored 3 sections of her bar, that's three-fourths. Now if I color the same amount she colored, I would have to color six sections, that's six-eighths. So that must be the answer.

$$\frac{3}{4} \times \frac{}{} = \frac{6}{8}$$

So now I have the first part of the equation, and the answer.

Now let's see. If I multiply the 3 by 2, I'll get 6. That's what I'm supposed to get. Now I'll have to multiply the denominator by 2 too, because if I don't, the fractions won't be equivalent. So that's what I did, and guess what! I did it right because the answer is what it's supposed to be: six-eighths.

$$\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$$

And then I colored six sections of my fraction bar green.

Reader, please color the six sections for Kayla.

As you can see, I colored the same amount as Ms. Gibbs' did. Three-fourths is equal to six-eighths. Hey, I made my first equivalent fraction and it wasn't hard at all! I just had to sit straight up in my chair so I could think better.

“Very good,” Ms. Gibbs said, “You multiplied each number in the fraction by two, and your answer was six-eighths. As you can see, multiplying made the numbers in the fraction *bigger* without changing its value.

“Now let’s divide each number in the fraction six-eighths by two:

$$\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

“Dividing made the numbers in the fraction *smaller* without changing its value.

“Kayla, any time you can make a fraction smaller without changing its value, that’s what you should do. It’s called *simplifying*. Three-fourths is the simplified form of six-eighths. It’s in its lowest terms.

“Now I’d like to see you simplify two other fractions: four-sixths and three-fifteenths.”

For the first fraction I saw I could divide each number by two, so that’s what I did, and here’s what I got:

$$\frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

It wasn’t so hard. For the next fraction I could see that dividing by two wouldn’t work, so I tried dividing by three. It worked, and here’s what I got:

$$\frac{3 \div 3}{15 \div 3} = \frac{1}{5}$$

“You did a good job simplifying those fractions. Next time we’ll talk about other ways to simplify,” Ms. Gibbs said.

“Now that you understand about equivalent fractions, you are ready to learn how to add and subtract unlike fractions.”

I guess I’m ready to learn how to do it. I hope it’s not too hard.

Chapter 5

Adding and subtracting unlike fractions

"Kayla," Ms. Gibbs said, "You can't really add unlike fractions just the way they are. You'll first need to find a common denominator."

I can't really add unlike fractions? A *common denominator*?   Oh, no, this is starting to sound too complicated already.

"You see, Kayla, the denominator not only tells you how many parts something is divided into, it also gives a *name* to what you're adding or subtracting."

"Huh?"

Ms. Gibbs continued to explain, "When we add one-third plus two thirds, *third* is the *name* of what we're adding. It's like adding one kitten plus two kittens: *third* is the name of something just like *kitten* is the name of something.

"However, if you wanted to add one kitten and one puppy you would first need to find something that is common to both. You could say one kitten plus one puppy equals two *animals*. Being an animal is what they have in common. Do you see that, Kayla?"

I shrugged my shoulders a little. Everyone knows that kittens and puppies are animals.

"Well, it's the same in math." Then Ms. Gibbs wrote this equation:

$$\frac{1}{2} + \frac{1}{4}$$

“Just like you couldn’t add kittens and puppies until you found something they have in common, you can’t add halves and fourths until you find something *they* have in common. But instead of finding a *noun* that’s common to both, like *animal*, you need to find a *number* that’s common to both denominators. Do you understand, Kayla?”

I didn’t answer. I was trying hard to pay attention to what Ms. Gibbs was saying. I wanted to ask a question, but I didn’t know what to ask.

Ms. Gibbs must have read my mind, because she said, “I know it sounds complicated, but things always do before you actually start doing them. After we work on a couple of problems, you’ll see that it won’t be too hard for you.

“Now, there is a short-cut way to find a common denominator that is pretty easy. And then there is another way that is a little longer and may seem just a *little* harder. Shall I start with the easy way?”

“Oh, yes,” I said, “Maybe the easy way will be easy enough for me.”

“OK,” Ms. Gibbs said, “Let’s use the example we’ve started with. Suppose you want to add the fractions one-half and one-fourth:”

$$\frac{1}{2} + \frac{1}{4}$$

“The short-cut way to find a denominator that is common to both fractions is simply to multiply the two denominators together. Let me do that:

$$2 \text{ x } 4 = 8$$

"Eight will be the new denominator for both fractions. Are you following me, Kayla?"

I nodded my head. I could see what Ms. Gibbs did. She simply multiplied the old denominators together and got eight. But then…

"Now, Kayla, the next step is to make equivalent fractions for our two original fractions. Since we changed the denominators for both fractions, we have to change the numerators or else…or else what, Kayla?

"Hey, I know that! That's because if we don't, the new fractions we're adding won't be equivalent to the old ones."

"That's right!" Ms. Gibbs said with a smile. "So that's the next step: Make equivalent fractions. And the final step – the easy step – is just to add the like fractions.

"We know that the new denominator for both fractions is eight. You just have to find a new *numerator* for each one."

Hmm. I was listening to and understanding what Ms. Gibbs was saying – well, sort of – but…but I wasn't so sure of what to do next.

I was glad to see Ms. Gibbs set up the first equation for me:

$$\frac{1}{2} \times \frac{\quad}{\quad} = \frac{\quad}{8}$$

Then she said, "Now you'll just need to decide what to multiply the original numerator by so that the new fraction is equivalent to the old one. Remember, you're not changing the *value* of the fraction, just the numbers."

Hmm…

I could see that the denominator was multiplied by 4 to get 8. Now I have to multiply the *numerator* by 4 also. If I don't, the new fraction won't have the same value as the old one. Sooo…

$$\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$$

That means one-half and four-eighths are equivalent fractions. And that means they have the same value.

Then Ms. Gibbs set up the second equation:

$$\frac{1}{4} \times \frac{}{} = \frac{}{8}$$

I could see that the denominator was multiplied by 2 to get 8, so I had to multiply the numerator by 2, too. Here's what I got:

$$\frac{1}{4} \times \frac{2}{2} = \frac{2}{8}$$

which means one-fourth and two-eighths are equivalent fractions.

There! I was all done, so I put my pencil down.

"Oh, you're not done yet," Ms. Gibbs said, "You didn't add the fractions."

So I added them and this is what I got: That part was easy!

$$\frac{4}{8} + \frac{2}{8} = \frac{6}{8}$$

“That’s good, Kayla,” said Ms. Gibbs, “Now simplify your answer.”

I did. That was easy, too:

$$\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

Ms. Gibbs continued, “Now let’s add these same two fractions again. But this time we’ll use the other method. It may seem a little harder at first, but you’ll see that in this problem it will actually be a little easier - and a little faster.”

“But…but Ms. Gibbs, I like the easy way. The easy way was easy enough for me. Pleeease, can’t we just do it this easy way?”

Ms. Gibbs didn’t pay any attention to my pleading. She just started explaining that harder way.

“Let’s start again with one-half and one-fourth. You’ll still need to find a common denominator as before, but this way will help you find the *lowest* common denominator. You’ll see why we do this later. By the way, “lowest common denominator” is often referred to by just its initials: LCD.

“The first thing you do is find multiples of each denominator. As you know, a multiple of a number is just the answer you get when you multiply that number by another whole number.

“That’s exactly what you did when you found the common denominator the short-cut way. You multiplied two times four and got eight, which is a multiple of both two and four. But as you will soon see, that common denominator might not be the *lowest* one.”

“Huh?” I asked.

"This will be easier to understand if we work together," she said. Let's find some multiples of the denominators in one-half and one-fourth."

"Ms. Gibbs, can't I just do it the easier way? This is starting to sound *way* too hard! Pleeease?"

"Kayla, *please* just let me finish. After you understand both ways, you can choose whichever way you want to do it.

"When finding multiples, I always start with the smaller denominator. So we'll start with two. You know your two times tables, so this will be very easy for you. I'll ask you your two times tables, and you give me the answers, OK?" Ms. Gibbs said.

"OK." And that's what we did. She asked, I answered, and this is what we got:

$$2 \text{ x } 1 = 2$$
$$2 \text{ x } 2 = 4$$
$$2 \text{ x } 3 = 6$$
$$2 \text{ x } 4 = 8$$

"OK, that's enough," Ms. Gibbs said, "Now let's find some multiples of four."

Again she asked and I answered, and this is what we got:

$$4 \text{ x } 1 = 4$$
$$4 \text{ x } 2 = 8$$

But then Ms. Gibbs stopped and said, “We don’t have to go any further, because we already have two multiples common to both denominators, four and eight. Since we want the *lowest* common denominator, the LCD, we’ll use four. Please circle the four in both lists of multiples.”

Reader, please circle the fours for Kayla.

“Now four is the new denominator for both fractions. The next step is to make the new fractions equivalent to the original ones. And the final step is just to add them!”

I saw that I didn’t have to change the one-fourth because it already had a four in its denominator. But I did have to make an equivalent fraction for one-half, so that’s what I did:

$$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$$

And then I added the two fractions:

$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

“Very good, Kayla!” Ms. Gibbs said. “Now let’s look at what you did. You figured out a common denominator two different ways and you ended up with these two equations:”

$$\frac{4}{8} + \frac{2}{8} = \frac{6}{8}$$

$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

“The first way you did it you added eighths, and you got six of them. The second way you did it you added fourths, and you got three of them.

“Some students make the mistake of adding the denominators, too. Just remember, denominators tell you the *name* of what you’re adding. In this example the names are ‘eighths’ and ‘fourths.’ Denominators should *never* be added.”

Then Ms. Gibbs asked, “Are the two answers equivalent?” Somehow I figured out in my head that they were, so I nodded.

“Yes, they are equivalent,” said Ms. Gibbs, “and both answers are correct. However, the second answer, three-fourths, is in a simpler form, and that makes it a better answer. You should always give the answer in its simplest form.

“Remember, in the second way of finding a common denominator, you circled the lowest common multiple and used that as the lowest common denominator. When you do that, the answer frequently comes out in a simpler form than when you use the first way, but it is not necessarily in its *simplest* form.

“There are other ways to work with unlike fractions, but this way helped me understand it the best. I hope it will be best for you, too.

I just said, “Hmm…”

“Kayla, we’ll talk more about simplifying fractions next week,” Ms. Gibbs said, “It’s getting late and I want you to work just two more problems before it’s time for you to leave.”

Ms. Gibbs asked me to add these two fractions:

$$\frac{1}{5} + \frac{1}{10}$$

At first I was just going to multiply the denominators together – you know, the easy way – but then I saw right away what I could use for a common denominator. It was ten. I didn’t even have to write anything down. I saw that ten is a common multiple, because two times five is ten. And it has to be the lowest because it is already one of the denominators!

So that’s what I used for my lowest common denominator, and this is what I got:

$$\frac{1}{5} + \frac{1}{10}$$

$$\frac{1}{5} \ \text{x} \ \frac{2}{2} = \frac{2}{10}$$

$$\frac{2}{10} + \frac{1}{10} = \frac{3}{10}$$

I looked up at Ms. Gibbs and she said, “That’s very good, Kayla. And your answer is in its simplest terms.”

I amazed myself! I think Ms. Gibbs was amazed, too. Somehow I knew right away what to use for my LCD. Hey, I must be getting math savvy!

Reader, did you know right away? If not, keep working these problems and you'll get more math savvy, too!

“This is the last problem for today,” Ms. Gibbs said.

Oh good! My head is getting too full.

“Imagine a big vase filled with colored marbles. One-sixth of the marbles is red, three-fifths of the marbles are green, and the rest are white. What fraction of the marbles is white?” Ms. Gibbs asked.

Hmm. That's an addition and subtraction problem. First I'll add the red and green marbles.

$$\frac{1}{6} + \frac{3}{5} =$$

These fractions are unlike so I have to find a common denominator. I think I'll try the little harder way because I don't think it will be too hard for me. Ms. Gibbs said that I should start with the smaller number so I listed the multiples of five first:

$$5, 10, 15, 20, 25, 30, 35$$

I think that should be enough.

Then I listed the multiples for six. They were:

6, 12, 18, 24, 30

And then I stopped. I could see that 30 was a common multiple so I circled both of them.

Reader, please circle the common multiples for Kayla.

Hey, I could have just multiplied the denominators together! But if I did that I wouldn't be sure it was the *lowest* common denominator, now, would I?

Now I have thirty for the denominator for each fraction. The next step is to make equivalent fractions.

I thought I could set up the equations myself this time and so I did. And I solved them too:

$$\frac{1}{6} \times \frac{5}{5} = \frac{5}{30} \quad \text{and} \quad \frac{3}{5} \times \frac{6}{6} = \frac{18}{30}$$

The next part is the easy one. I just need to add the like fractions:

$$\frac{5}{30} + \frac{18}{30} = \frac{23}{30}$$

Now I know that twenty-three thirtieths is the fraction of the marbles that are red and green. But the problem asks what fraction of the marbles is white. Well, if the marbles are not red or green they must be white because there are no other colors in the problem. So now I need to subtract twenty-three thirtieths from one whole vase of marbles and that's what I did:

$$\frac{30}{30} - \frac{23}{30} = \frac{7}{30}$$

So the fraction of all the marbles in the vase that are white is seven-thirtieths!

I did it and I think it's right. Ms. Gibbs thought so too, because she said, "That's very good, Kayla. You did everything right."

She then took a piece of paper from her folder and said, "Here is a paper with a 5 x 6 grid on it for you to take home.

"Have the boxes represent the marbles in the jar. If you color in the red and green boxes, what is left uncolored will represent the white marbles.   Doing this may help you visualize your answer."

And that night, after supper, that's what I did! It was a little tricky but I did it and I'm pretty sure I did it right!

Reader, the grid Ms. Gibbs gave Kayla is on the next page. Please color in the boxes so you can visualize the answer too. When you finish, look on page 50 to see how Kayla did it.

Reviewing what I learned

There is lots of stuff to review so let's get started.

I hope you practiced your nine times tables and were able to put all of them into your grid. The short cut for the eleven times tables is so easy, you won't even have to practice them. You just double whatever digit you're multiplying eleven by. It's that easy. Oh, that only works until nine. Make sure you read what I wrote about the eleven times tables on pages 12 to 14 so you'll understand it.

Now on to equivalent fractions. These are fractions that have different numbers but they have the same value. I know it seems like a paradox. It did to me at first, but now that I understand it, it doesn't any more.

And I figured out why I had so much trouble with it at first. You see, if you multiply any whole number by one, that whole number doesn't change, and neither does its value. Watch, I'll show you:

$$\mathbf{2 \times 1 = 2}$$

See, the two doesn't change and neither does its value.

But when I multiply

$$\mathbf{\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}}$$

the two-over-two is the same as one, so when I multiply one-half by it, I don't change the value of the fraction. But I DO change the numbers. And that's what was so confusing for me.

If you have fraction bars and you draw in an extra line, like Ms. Gibbs did on page 22, you'll see how one-half and two fourths have the same value even though they have different numbers.

And if you want to see how two-thirds of a pizza is the same as four-sixths of a pizza, look on pages 23 and 24.   I drew those two extra dotted lines by myself to help me see it. Did you see it too?

Make sure you really understand about equivalent fractions because if you don't, you won't be able to add unlike fractions.

Adding and subtracting them can be very tricky, so I wrote out the steps for myself and for you. Now, if you follow these steps - and you practice…you *have* to practice - you should be able to do this. Just follow along with me:

Step 1. Find a common denominator. A common denominator is really just a number that is a multiple of both denominators.

Now there is an easy way to find a common denominator and one that seems a little harder. The easy way is just to multiply the original denominators together. The answer has to be a common denominator, even though it might not be the *lowest* common denominator.

Another way to find a common denominator is to list the multiples for each denominator. You find multiples of a number by multiplying it by one, then two, then three, and so on. Just do this for both denominators until you find the first multiple that is the *same* for both. Then circle them both so you don't get mixed up. That's what I did on pages 35 and 36.

Got it? Now on to Step 2.

Step 2. Make equivalent fractions using the new denominator to find the new numerators. If you don't do this step, the fractions you're supposed to add or subtract won't be equivalent and then the answer you get will be all wrong.

Step 3. This is the easy one. You'll have like fractions now, so you can just add or subtract the numerators to get the correct answer. And remember, you NEVER add the denominators!

It seems like a lot of work but if you practice to get it down-pat, it's not. So practice, practice, practice. That's what Cleveland told me I have to do to get better at basketball. And that's what I'm telling you *you* have to do to get better at math!

Just don't get discouraged. This is hard stuff. Ms. Gibbs told me this stuff was hard for her, and if it was hard for her, you know it must be really hard.

Practice Problems

1. Reduce these fractions:

1a. $\frac{2}{4}$ = 1b. $\frac{4}{6}$ = 1c. $\frac{3}{15}$ =

1d. $\frac{4}{12}$ = 1e. $\frac{2}{20}$ = 1f. $\frac{20}{30}$ =

1g. $\frac{8}{44}$ = 1h. $\frac{7}{21}$ = 1i. $\frac{10}{24}$ =

2. If any fraction below is not in its simplest form, simplify it:

2a. $\frac{2}{6}$ = 2b. $\frac{1}{5}$ = 2c. $\frac{2}{8}$ =

2d. $\frac{2}{4}$ = 2e. $\frac{3}{9}$ = 2f. $\frac{9}{27}$ =

3. List the multiples for each denominator in these pairs of fractions. Then circle the lowest common multiple for each pair:

3a. $\frac{1}{5}$ and $\frac{1}{10}$

3b. $\frac{1}{6}$ and $\frac{1}{4}$

3c. $\frac{1}{3}$ and $\frac{1}{8}$

3d. $\frac{1}{2}$ and $\frac{1}{5}$

3e. $\frac{1}{4}$ and $\frac{1}{5}$

3f. $\frac{1}{5}$ and $\frac{1}{7}$

4. Are the pairs of fractions below equivalent? Answer Yes or No. See if you can first figure it out in your head like I did on page 35.

4a. $\frac{2}{10}$ and $\frac{4}{20}$ ______

4b. $\frac{1}{3}$ and $\frac{2}{3}$ ______

4c. $\frac{2}{4}$ and $\frac{4}{8}$ ______

4d. $\frac{1}{7}$ and $\frac{3}{21}$ ______

4e. $\frac{1}{5}$ and $\frac{5}{10}$ ______

4f. $\frac{1}{4}$ and $\frac{3}{12}$ ______

4g. $\frac{6}{8}$ and $\frac{3}{4}$ ______

4h. $\frac{2}{6}$ and $\frac{1}{3}$ ______

5. Solve these equations. Watch the signs! Simplify if you can:

5a. $\frac{1}{4} + \frac{3}{4} =$

5b. $\frac{2}{5} \times \frac{3}{5} =$

5c. $\frac{6}{8} - \frac{2}{8} =$

5d. $\frac{4}{9} - \frac{1}{9} =$

5e. $\frac{5}{7} - \frac{2}{7} =$

5f. $\frac{3}{8} + \frac{1}{8} =$

5g. $\frac{10}{12} + \frac{6}{12} =$

5h. $\frac{3}{5} \times \frac{2}{5} =$

6. Add or subtract these unlike fractions. Simplify if you can:

6a. $\frac{3}{5} - \frac{1}{3} =$

6b. $\frac{8}{12} + \frac{1}{4} =$

6c. $\frac{2}{10} + \frac{1}{2} =$

6d. $\frac{3}{4} - \frac{1}{2} =$

6e. $\frac{5}{12} - \frac{1}{4} =$

6f. $\frac{2}{11} + \frac{1}{22} =$

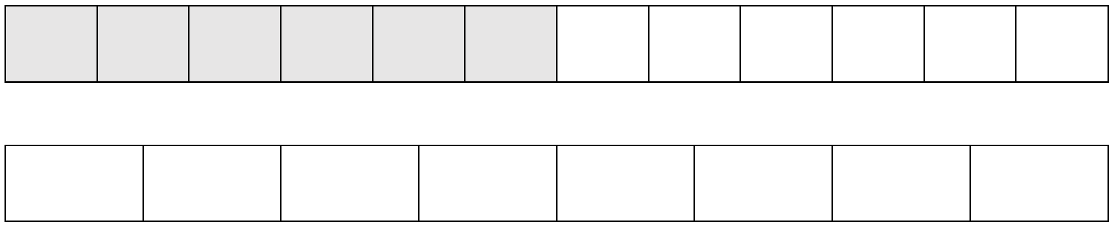
6g. $\frac{5}{6} - \frac{1}{3} =$

6h. $\frac{2}{3} + \frac{3}{4} =$

6i. $\frac{6}{8} - \frac{1}{3} =$

6j. $\frac{4}{7} - \frac{1}{2} =$

7. Here are two fraction bars; one divided into twelve sections, the other divided into eight sections:



As you can see, six sections in the first fraction bar are shaded. What fraction of the second fraction bar would you need to color so that it equals what is shaded in the first? Do the math first, then color that many sections with your favorite color.

8. There is a big jar of marbles. One-third of them is yellow, one-fourth is purple and the rest of them are black.

What fraction of the marbles in the jar is black? _________

Here is how Kayla colored in the boxes in the grid on page 42:

R		G	G	G
R		G	G	G
R		G	G	G
R		G	G	G
R		G	G	G
		G	G	G

R stands for red and G stands for green, and the rest are white. Remember that each box is one-thirtieth of the total. The way that Kayla colored in the boxes makes it easy to see that the fraction of white marbles is seven-thirtieths.

There are other ways you could have colored in the boxes. The important thing is that there must be five red, eighteen green and seven white boxes.

Something extra

The word “paradox” is from the Ancient Greek language. It’s made up of two other words: “para” and “dox.” “Para” means “contrary to,” and “dox” means “opinion,” and so it means contrary to what you would think. Isn’t that a neat word? At first, having fractions with the same value but with different numbers sure seemed like a paradox to me, but now that I understand it, it doesn’t.

Answers

1a. $\frac{1}{2}$

1b. $\frac{2}{3}$

1c. $\frac{1}{5}$

1d. $\frac{1}{3}$

1e. $\frac{1}{10}$

1f. $\frac{2}{3}$

1g. $\frac{2}{11}$

1h. $\frac{1}{3}$

1i. $\frac{5}{12}$

2a. $\frac{1}{3}$

2b. $\frac{1}{5}$

2c. $\frac{1}{4}$

2d. $\frac{1}{2}$

2e. $\frac{1}{3}$

2f. $\frac{1}{3}$

3a. 5, 10
10

3b. 4, 8, 12
6, 12

3c. 3, 6, 9 ,12, 15, 18, 21, (24)

8, 16, (24)

3d. 2, 4, 6, 8, (10)

5, (10)

3e. 4, 8, 12, 16, (20)

5, 10, 15, (20)

3f. 5, 10, 15, 20, 25, 30, (35)

7, 14, 21, 28, (35)

4a. Yes 4b. No 4c. Yes 4d. Yes

4e. No 4f. Yes 4g. Yes 4h. Yes

5a. 1

5b. $\frac{6}{25}$

5c. $\frac{1}{2}$

5d. $\frac{1}{3}$

5e. $\frac{3}{7}$

5f. $\frac{1}{2}$

5g. $\frac{4}{3}$

5h. $\frac{6}{25}$

6a. $\frac{4}{15}$

6b. $\frac{11}{12}$

6c. $\frac{7}{10}$

6d. $\frac{1}{4}$

6e. $\frac{1}{6}$

6f. $\frac{5}{22}$

6g. $\frac{1}{2}$

6h. $\frac{17}{12}$

6i. $\frac{5}{12}$

6j. $\frac{1}{14}$

7. $\frac{6}{12} = \frac{1}{2}$ So color one-half of the second fraction bar. Half of eight is four boxes to color.

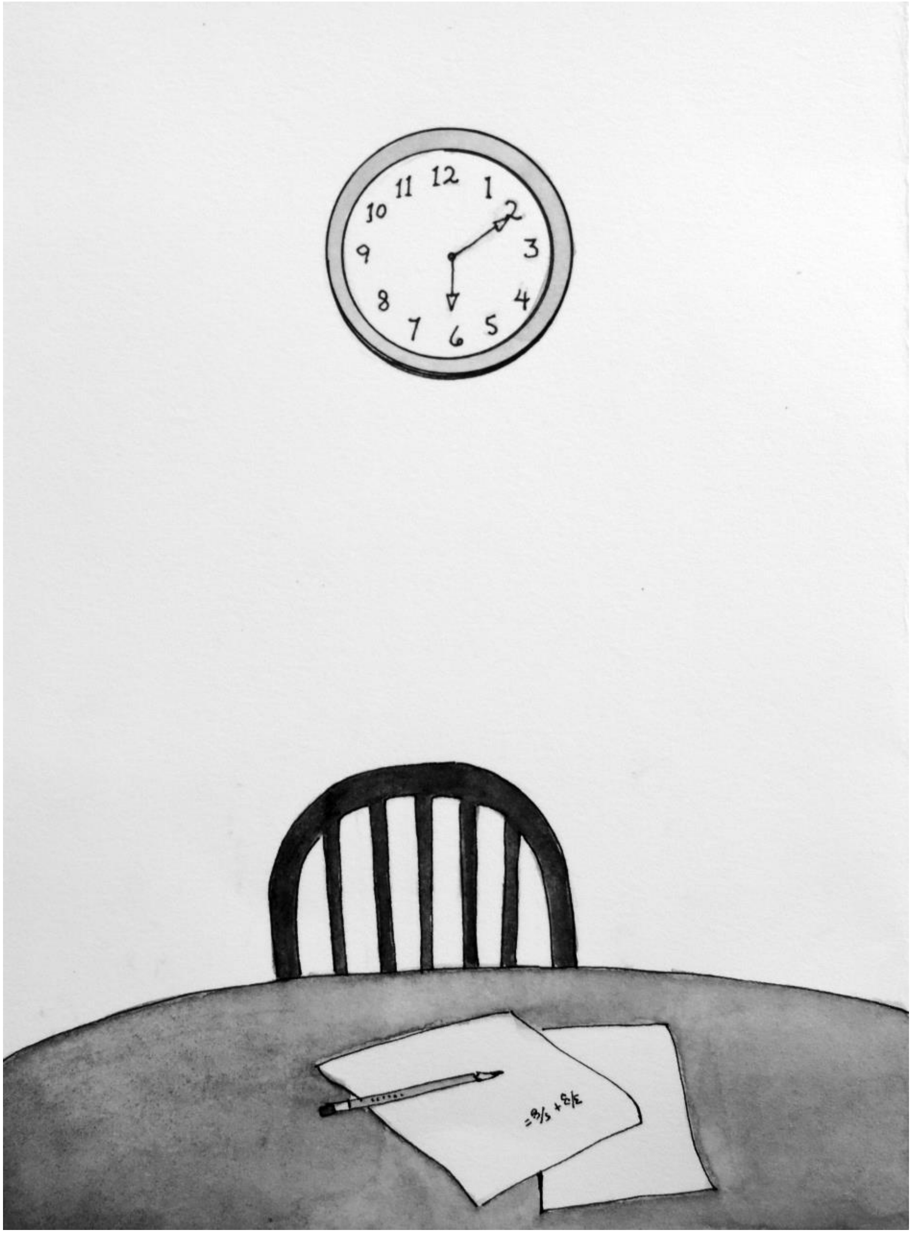
8. $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$ are yellow and purple.

$\frac{12}{12} - \frac{7}{12} = \frac{5}{12}$ are black.

About Tutoring Math

Kayla is like many kids you may tutor; she's a reluctant learner. In this book, she first showed her reluctance by thinking it was time to go back to her classroom before Ms. Gibbs had completed the lesson. Then in Chapter 4 she began to slouch. Engaging a reluctant learner with a question sometimes helps, as Ms. Gibbs did on page 32 when she asked, "or else what, Kayla?"

Deciding how much material to cover in one sitting is a judgment call. If you're tutoring from this book, you may want to cover the material in two, three, or perhaps more sessions. In the long run, it really doesn't matter how long it takes for students to learn the material. What really matters is how well they learn it.



Hey! Where's Kayla? You'll find out in Book 6