Learning Math with Kayla

Book 10: Working with decimals and per cents

Vicki Meyer

Illustrator Sue Lynn Cotton

The Learning Math with Kayla Books

Book 1 Adding and subtracting like fractions
Book 2 Multiplying fractions
Book 3 Learning multiplication facts
Book 4 Place values, Multiplying large numbers
Book 5 Adding and subtracting unlike fractions
Book 6 Learning about improper fractions and mixed numbers
Book 7 Dividing fractions
Book 8 Adding and subtracting large numbers
Book 9 Solving long division problems
Book 10 Working with decimals and per cents
Book 11 Learning about negative numbers
Book 12 Problem solving!

Website: **www.learningwithkayla.org**

Gedanken Publishing
Berea, Ohio

About the Kayla Books

The Kayla books tell the story of a fourth-grade girl who has gotten so far behind in her math class that she is not able to understand what her teacher is trying to teach her. Her math teacher, Mr. Williams, is aware of how poorly Kayla is doing. He decides a tutor would be the best way to help Kayla learn her math.

In this tenth book, Ms. Gibbs teaches Kayla about decimals and per cents. Kayla learns how to convert fractions into per cents and to do calculations involving per cents; for example, finding the sales tax on an item bought in a store. She also learns how to do long division when the divisor is a decimal, and how multiplying and dividing by powers of ten can be done simply by moving the decimal point.

There are twelve books in this series. Whether you're a fourth grader, in middle school or in high school; a Mom or Dad or a Grandparent, you can learn along with Kayla.

The story is told by Kayla, right before she goes off to college.

About Kayla

I have been asked if Kayla is a real person. She and others in the books are composites of the many kids I have tutored, plus friends. I have spent time in El Salvador and witnessed the stark inequality and poverty that most people in that country live with. That's why I had Kayla's friend, Luz, come from El Salvador. Luz is the name of a good friend of mine from Mexico. She is helping me with the Spanish in this book.

The Author

After Vicki raised six really smart kids, she began studying for her Ph.D. in order to keep up with them. She taught at the university level for 25 years, then began tutoring elementary school students. Vicki soon found a new career for herself, tutoring math for at-risk kids, writing about her experiences, and putting together the Kayla books. You can contact Vicki by going to the website (**www.learningwithkayla.org**) and clicking on "Contact."

Vicki lives with her husband, Ed, in Sarasota, FL.

Acknowledgements

Thanks to Charles Daniel for the excellent job he does in proofreading the text. His expertise in mathematics and grammar ensures that both kinds of errors are minimized in the Kayla Books.

And a special thanks to my husband, Ed, for all of his great suggestions, his skillful editing, and especially his patience. I would not be able to complete the books without him.

DEDICATION

To my mother, Phyllis Hurtova, who was prevented from going past the fourth grade due to political unrest in Czechoslovakia, but continued to be a life-long learner.

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Chapter 1

Luz, Elizabeth - and me

We have a new girl at our school and guess what! She doesn't speak any English. Now, she might understand a little tiny bit of English, but she speaks only Spanish. And do you know why? She's from El Salvador and in El Salvador, that's what people speak - Spanish. Her name is Luz.

But wait! I better tell you how to pronounce her name. It's pronounced "loose," just the way I like my pajamas. When I go to sleep, I don't like my pajamas too tight on me. I like them loose, and that's how you pronounce the name of the new girl in our class. It's spelled L-u-z but it's really pronounced "loose." Got it?

And I want to tell you about another girl in our class. Her name is Elizabeth and she can speak Spanish too. She doesn't speak Spanish real fast like Luz does though, and it doesn't really sound the same - not really - but Luz can understand her – well, at least some of the time she can.

So here is how it goes: Elizabeth speaks English and a little bit of Spanish, Luz speaks only Spanish. Did you remember how to pronounce Luz's name? Here's a hint, just think of my pajamas. Oh, and I speak only English.

Do you remember Elizabeth? She's one of the really smart girls in our class. In fact, she's probably the smartest person in our whole class - except for Mr. Williams, of course. I told you a little about her a while ago. I'll tell you again.

A long time ago, Cleveland kicked over Elizabeth's chair - but he really didn't mean to. You see, Cleveland was bored in our math class and so he started kicking the chair in front of him - and it was Elizabeth's chair. Well, she didn't like it so she just stood up. And that's when the chair fell over and it made a whole lot of noise.

If you don't remember, here's a picture of Elizabeth with her hands on her hips. Oh, that's Cleveland in the picture too.

Reader, if you want, you can reread the whole story about Elizabeth and Cleveland. It's in Book 2.

Well, anyway, Mr. Williams got real mad and sent Cleveland to the Time-out room. He sent me there too but not because I knocked someone's chair over. It was because I was drawing pictures in my math class and I wasn't supposed to. Now do you remember?

Now back to the story.

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We're having a really, really big test next week and Mr. Williams wants to make sure we know all about fractions. Right before lunch, he reviewed how to divide them.

It's lunch time now and I decided to have lunch with Luz. You see, she doesn't have many friends in our school because she's new. I'm not new but I don't have many friends either so I thought maybe we could be friends together.

As I entered the cafeteria, I looked around for Luz. I saw her across the room and I went to sit down next to her. Well, Luz was already sitting with someone; it was Elizabeth! I turned to go but Elizabeth said, "Why don't you sit with us?"


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There was room next to Luz so I sat down next to her.

Elizabeth had her notebook out and was teaching Luz fractions. She was teaching her how to divide them. Luz was speaking only in Spanish but Elizabeth was speaking in Spanish *and* in English. I wasn't speaking in anything. I was just listening. I heard Elizabeth say, "Don't ask why, just flip the divisor and multiply."

Luz kept saying in Spanish, "Por qué? Por qué?" and this is what Elizabeth said, "Porqué por división no preguntes 'por qué,' solomente - uh, uh - ponéal revés la segunda fracción.

Elizabeth explained to me what she was saying to Luz: "I'm trying to explain about dividing fractions. Luz wants to know why she's supposed to flip the second fraction over and I told her not to ask why, just flip it over and multiply.

"That's a rhyme that I learned in summer camp," Elizabeth explained with a little smile. And then she added, more serious-like, "I really don't know why flipping the divisor over works, but I know it works - so I just do it."

“Well, Elizabeth, it’s an algorithm,” I replied.

“An algorithm?” Elizabeth asked. She looked surprised.

“Yes, an algorithm. An algorithm is just a simple rule. Anyone who wants to divide by a fraction can do it by using this rule - which is just: ‘invert the divisor and multiply.’ This rule works every time you divide fractions!” I explained.

She was quiet for a minute and then said, “Why don’t you just call me Liz. All my friends do.”

And so that’s what I did!

Chapter 2

Liz and Me

I guess Elizabeth, oh, I mean Liz, is my friend because - well - because she said she was - well she sorta did!

As we walked back to our classrooms together, she asked me where I learned about algorithms. That’s when I told her all about Ms. Gibbs.

Liz is real, real smart. She knows so many things, so I asked her where she learned all she knows. She explained, “Oh, every year I go to summer camp at the same time my mom and dad take their vacation. I learn a lot of things there.”

I never went to a summer camp but I thought I knew something about it, so I said, “I thought summer camp just had camping stuff to do, like ... uh, well like making camp fires.” I remembered reading about that once.

“Well we do have campfires but there is lots of other stuff we do at camp too. You see, the camp I go to is an *enrichment* camp,” Liz explained.

And then she continued, “That means we have school stuff to do, like math and writing, and learning an international language. We can choose whatever language we’d like to learn. I chose Spanish, although I’m still not very good at it.” Elizabeth’s voice trailed off when she said this.

“And then we have camping things to do, like swimming, sailing, basketball and yes, campfires too,” Liz explained.

Now it was my turn to be surprised!

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I didn't know what to say, so I just said, "Oh." I was thinking about all those things Liz did at camp. Why, she even learned how to play basketball. Hmm, I wonder if she knows how to dribble.

I was trying to think of something to say but I couldn't think of anything. We were both silent for a bit and it seemed to me a bit awkward. Oh, if only I could think of something to say.

Finally, I thought of something: "Liz and Luz are almost the same, except for just one letter." Oh, no, as soon as I said that, I realized how dumb it sounded.

But Liz replied, "Yes, but Luz means "light" in Spanish; Liz doesn't mean anything, it's just short for my name, Elizabeth.

Oh! Kayla doesn't mean anything either, it's just my name. And then I added, "My dad named me."

“Well, your dad has good taste in names. Kayla is a very pretty name,” Liz said as she turned to go to her next class.

And then she said, “See you later, Kayla.”

And I said, “See you later, Liz.”

Chapter 3

Ms. Gibbs and her map

I’m so glad it’s tutoring day because I’ve been waiting to tell Ms. Gibbs about Luz. After we greeted each other, I was thinking of all the things I was going to tell Ms. Gibbs about Luz - but I soon realized I didn’t know very much.

I know she’s from El Salvador but I don’t know where that is or why she came here, or even how she got here. I bet Ms. Gibbs knows. Hey, I’ll have her guess!

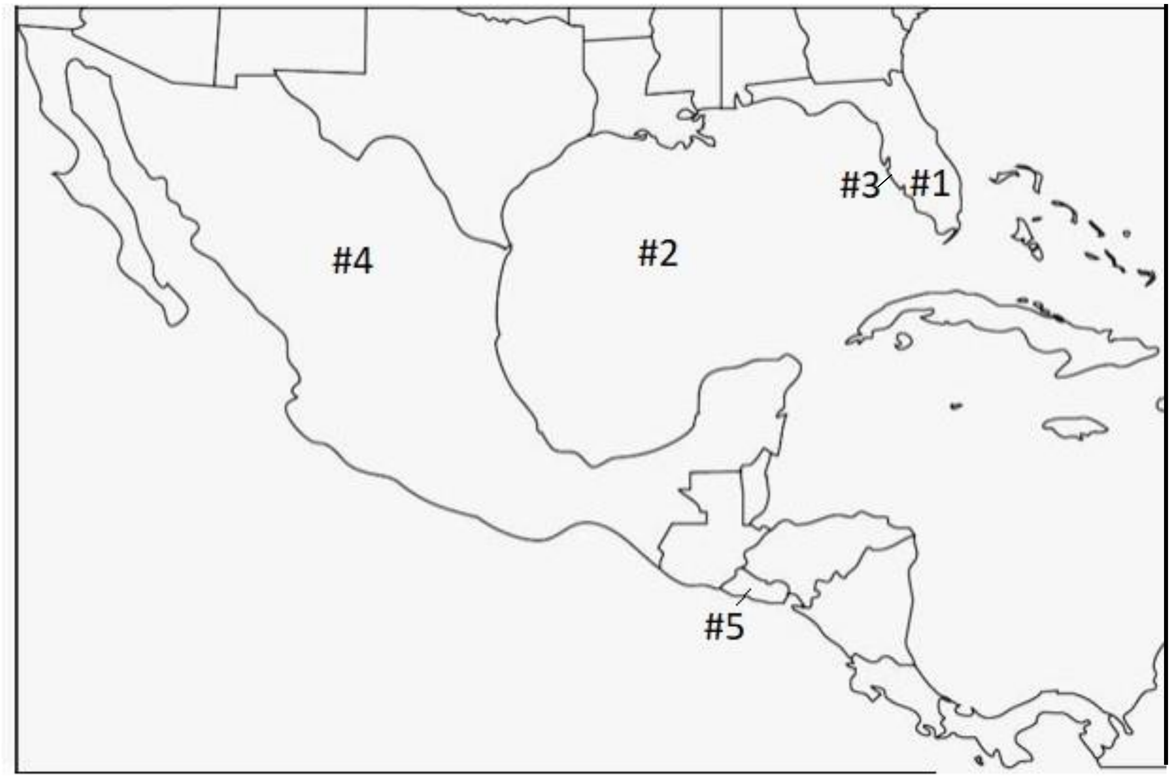
So I asked her this: “Ms. Gibbs, do you know of any country where people speak Spanish? Because if you do, that’s the country Luz came from. You see, she’s the new girl in our class and she speaks only Spanish.”

“Kayla, there are many countries in the world where people speak Spanish,” Ms. Gibbs explained. “I happen to know that Luz, the new girl in your class, is from El Salvador.”

As Ms. Gibbs said that, she took a map from her folder. It looked old. I noticed it was taped up in a couple of places.

As soon as I saw the map I said, “We have a map like this at home, but it’s much newer.” Oh no, I realized I shouldn’t have said that. Ms. Gibbs might feel bad that her map is so old.

But Ms. Gibbs said, “This map looks old but it’s really not very old. It’s just that I use it a lot. You see, I keep it right by the television set. If I’m reading the newspaper or watching the news and I don’t know where a country is that’s in the news, I just look for it on my map. In that way, I have a better understanding of the region being discussed,” Ms. Gibbs explained.



#1 Florida #2 Gulf of Mexico #3 Sarasota

#4 Mexico #5 El Salvador

Reader, do you know where you live on a map? If you don't, have someone show you. And if you don't have a map at home, ask your librarian. She'll have one.

“Oh,” I said. Then I told Ms. Gibbs about my momma’s map. “My momma keeps her map in the bookcase. She showed me where Florida is and Sarasota too - on the map,” I explained to Ms. Gibbs. The map was open and I could see Florida.

It’s right on the water - right there. It’s a peninsula. That means it has water on three of its sides. Sarasota is a city in Florida and that’s where I live.

I continued looking at the map. “And I know what all this water is called, it’s called the Gulf of Mexico,” I said as I pointed to the Gulf of Mexico. “Sometimes, I play at a beach. And sometimes I go into the water, too, the Gulf water. But I always stay right here at the edge.”

Reader, #1 is Florida, #2 is the Gulf of Mexico, and #3 is the beach where Kayla plays. It’s right at the edge of Sarasota (on the map).

“Yes, that’s right,” Ms. Gibbs said, then she pointed to an area across the water. “And here is Mexico.

#4 is Mexico

“And right here is El Salvador,” Ms. Gibbs said as she pointed to El Salvador on the map. “That’s where Luz is from.”

and #5 is El Salvador

“It’s not very big, is it?” I asked.

“No, it’s not very big at all,” Ms. Gibbs replied. “Those who want to come to the United States from El Salvador must first travel through Mexico.”

Ms. Gibbs continued, “I don’t know how Luz got here, but right now there are large numbers of people coming to the United States. Some of them have cars or trucks but most of them walk – some walk the whole way. They stay together in large groups because they feel it’s safer.”

“Safer?” I asked.

“Yes, safer. The trip is dangerous. Some people have to walk through a desert, and cross a river. And there are rattlesnakes to watch out for too,” Ms. Gibbs explained.

But…but why would they leave their country?” I asked. “Wouldn’t they just rather stay at home where it’s safe? I know I would.”

“They may not feel safe at home and that might be why they’re leaving,” Ms. Gibbs replied. “Some families leave because they want a better life for their children. You see, if they stay, there is very little opportunity for them to make their lives better.”

Ms. Gibbs continued to explain about the trip Luz had to take. “It may take several weeks to walk from El Salvador to the United States, especially if there are small children. They stop only to eat, drink. And to sleep. It’s a dangerous trip. That’s why they travel in large groups. They feel it’s safer.”

“What do they eat and drink? Do they bring their own food?” I was really wondering where they go to the bathroom but I didn’t want to ask Ms. Gibbs that!

Ms. Gibbs explained “Well, some good people leave jugs of water along the road for them. There are also organizations that help too. They may bring them food and clean clothes – and encouragement, of course.”

Ms. Gibbs continued, “You see, in these countries where people are leaving, there is much inequality. While there are a few families with a lot of wealth, the rest of the people in the country are very poor - sometimes even without enough food to eat.   And anytime there is so much inequality, there is always violence.” I felt very sad when she said all this.

And then she said firmly:” “There is one thing I know for sure: if there was more justice and much less inequality, people would not want to leave their countries.”

As she put her map away, Ms. Gibbs said. “I’m so glad that Luz arrived here safely and you and she are becoming friends. But right now, it’s time to learn about decimals and per cents. We’ll start with decimals.”

Chapter 4

Decimals

Ms. Gibbs started by asking, “Kayla, do you remember when we were doing multiplication, we talked about place values and the importance of keeping numbers in the correct columns?”

I remembered that we talked about place values, but…but before I could answer, Ms. Gibbs continued, “We made a table with each place value column labelled,” and she took a paper out of her folder that looked like this:

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones

When I saw the table I remembered! If we start putting numbers in the “Ones” column, like 1, 2, 3, and so on, when we get to 9, we have to start over with zero, and put a 1 in the “Tens” column. The number in each column tells us how many of the name at the top of the column there are. For example, if we put the number 4,372 into the table:

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
			4	3	7	2

we know that the number of “Thousands” is 4, the number of “Hundreds” is 3, the number of “Tens” is 7, and the number of “Ones” is 2. We read the number as “Four thousand, three hundred and seventy-two.”

Ms. Gibbs said, “This table shows only numbers greater than one. We can extend it to the right to show numbers *less* than one.”

Oh-oh, I thought to myself, this is starting to get complicated.

She continued, “In this table, as we go from right to left, each column is ten times greater than the previous one: ‘Tens’ is ten times ‘Ones’, ‘Hundreds’ is ten times ‘Tens’, ‘Thousands’ is ten times ‘Hundreds’ and so on. I know this is a lot of words, but the idea is quite simple, don’t you agree, Kayla?”

“Well, sorta,” I said, but I wasn’t so sure I would use the word, “simple”!

I guess if you look at the table while you say these words, it’s not really that complicated after all. The headings of the columns get multiplied by ten when you go from right to left.

“Now, Kayla, I want you to think about this: If we wanted to add one more column at the *right* of this table, what would its heading be?”

I thought to myself, “Ten times this heading would have to be ‘Ones’ so it must be less than one.”

“Can a heading in this Place Value Table be a fraction?” I asked.

Ms. Gibbs said, “I can tell by the question you just asked me that you are following me. The fraction you are thinking of is one-tenth, isn’t it? Because ten times one-tenth is equal to one:

$$\mathbf{10} \times \frac{\mathbf{1}}{\mathbf{10}} = \mathbf{1}$$

“We have a special way of writing one-tenth as a decimal. We write it as **0.1** and the period before the **1** is called a ‘decimal point’. We don’t really need the zero before the decimal point, but using it avoids confusion.

“The heading for this column in a Place Value Table is “Tenths” and like all the other entries in the table, the digit in this column tells us how many tenths there are in the number in the table.

“If we include this new column, our table looks like this:

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones	Tenths

"And if we put a number into this table, for example:

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones	Tenths
			9	**4**	**8**	**2**	**.7**

"We would read it as 'Nine thousand four hundred eighty-two point seven,' where 'point' is short for the decimal point. Or we could say 'Nine thousand four hundred eighty-two and seven tenths.' Either way is accurate, because 'point seven' and 'seven tenths' are the same thing:

$$\mathbf{0.7} = \frac{\mathbf{7}}{\mathbf{10}}$$

"As you may have guessed, Kayla, we can keep adding columns to the right, with each one being one-tenth of the one to its left. The next one would be…"

"It would be one-tenth divided by ten!" I interrupted Ms. Gibbs. I wasn't sure what one-tenth divided by ten was, but I knew that had to be right, and it was.

"That's right, Kayla, and dividing by ten is the same as multiplying by one-tenth. So we have:

$$\frac{\mathbf{1}}{\mathbf{10}} \times \frac{\mathbf{1}}{\mathbf{10}} = \frac{\mathbf{1}}{\mathbf{100}}$$

"This fraction is one one-hundredth, and using decimal notation it is written **0.01**. The next column after 'Tenths' in a Place Value

Table has the heading, 'Hundredths'. Similar reasoning tells us the next column would be 'Thousandths,' with the fraction one one-thousandth written as $\mathbf{0.001}$ in decimal notation, and so on":

$$\frac{\mathbf{1}}{\mathbf{1000}} = \mathbf{0.001}$$

I had to ask Ms. Gibbs, "Aren't fractions good enough to talk about numbers less than one? Why do we need to use decimals, too?"

"Well, Kayla, there are things in everyday life that use decimals, so it is good to understand them. For example, money uses the decimal system. When we see $\$\mathbf{5.23},$ the $\mathbf{5}$ is in the 'Ones' column, the $\mathbf{2}$ is in the 'Tenths' column, and the $\mathbf{3}$ is in the 'Hundredths' column, where the column names refer to 'dollars': 'Tenths' means 'tenths of a dollar,' or dimes. 'Hundredths' means 'hundredths of a dollar,' or cents. We read $\$\mathbf{5.23}$ as 'five dollars and twenty-three cents.'

"Meter readings use the decimal system, too. Like the odometer in a car, or an electric meter on a house. So it's a good idea to understand both fractions and decimals – and per cents, too, which we'll talk about after we talk about decimals.

"Doing math with decimals requires a little more care than with whole numbers. For example, when we add or subtract decimals, we must continue to be careful about lining up place values. This means that numbers containing a decimal point must all have their decimal points in the same column.

"For example, to add $\mathbf{1.54}$ and $\mathbf{32.97}$, we would write:

$$\begin{array}{r} \mathbf{1.54} \\ \underline{\mathbf{32.97}} \\ \mathbf{34.51} \end{array}$$

Where you can see the decimal points are lined up."

"That's pretty easy to remember," I said.

Ms. Gibbs continued to explain: "When we multiply with decimals, we have to know where to put the decimal point in the product. This will be a new algorithm for you:

Count the total number of digits to the right of the decimal points in the two numbers you are multiplying together, and put the decimal point into the product exactly the same number of spaces from its right end.

"Hmm… That sounds kinda complicated," I said.

"Well, it may sound complicated at first, but as soon as we look at an example you will see that it's really quite simple to do.

"Let's multiply those same two numbers together. This will be good practice for you!"

"OK, I know I can do this. Just watch me, Ms. Gibbs!":

$$\begin{array}{r} {\scriptstyle 1\,4\,3\;\;} \\ {\scriptstyle 1\,3\,2\;\;} \\ 32.97 \\ \underline{\times 1.54} \\ 13188 \\ 164850 \\ \underline{329700} \\ 507738 \end{array}$$

"You got the numbers correct in the answer, but you haven't inserted the decimal point, Kayla. Remember that you must count the total number of digits to the right …"

"Oh, I remember, Ms. Gibbs!" I interrupted her because I didn't want her to think I forgot what she told me. "There are four digits to the right of the decimal points; two in 32.97 and two in 1.54. That means I have to move the decimal point four spaces to the left in my answer. My answer is 50.7738. And that makes sense, because 1.54 is a little more than 1.5, which is one and a half, so the answer should be *nearly* one 32.97 and a half of 32.97. But 32.97 is *nearly* 33, and half of 33 is *nearly* 16, so our answer should be *nearly* $33 + 16$, which is 49. And 49 is *nearly* the answer I got. That means I put the decimal point in the right place."

"That's right, Kayla! And another way of checking whether we put the decimal point in the correct place is to compare our answer with the answers we would have gotten if the decimal point were one place to the left or one place to the right. In the problem you just did, those two answers are 5.07738 and 507.738.

"Your math savvy tells you that multiplying a number close to 33 by a number greater than 1 cannot possibly result in a number close to 5," said Ms. Gibbs. "Also, your math savvy tells you that multiplying a number close to 33 by a number less than 2 cannot possibly result in a number greater than 500. This provides a quick check on where you put the decimal point in the answer."

"You were right, Ms. Gibbs," I said, "it really sounds much harder than it is. I hope dividing with decimals is just as easy!"

"Well, we'll talk about how to divide by a decimal after we look at how to convert a fraction into a decimal. I think you'll see it also is quite easy to do.

"You can convert a fraction into a decimal using the division we learned about last time. For example, what is the decimal that is the same as one-fourth? We just divide 1 by 4":

$$4\overline{)1}$$

"But Ms. Gibbs," I said, "When we did division, we said that if **4** doesn't go into **1** we have to include the next number, and there's no next number here."

"You're right, Kayla, but we can put a 'next number' after the **1**. We can think of every whole number as having a decimal point after it, with as many zeros after it as we want. In other words, we can write the whole number, **1**, as **1.0**, or **1.00**, or even **1.0000000** if we want to; they all equal the same thing: 'one.'

"Let me rewrite the division problem like this:

$$4\overline{)1.00}$$

"It's exactly the same problem, but now…"

"Now I can do the division," I said, "because **4** doesn't go into **1**, but it goes into **10**, so I put a **2** over the first zero and just do the long division problem like this":

$$\begin{array}{r} 25 \\ 4\overline{)1.00} \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

"That's very good, Kayla! I'm glad to see you remembered the algorithm for long division. But there is one new thing for you to learn when using decimals: **You must put a decimal point in the quotient exactly above the decimal point in the dividend.** When this is done, the answer is written **.25** and we say, 'point two five' or 'twenty-five hundredths.'

“This shows that the decimal equivalent of ‘one-fourth’ is ‘point two five’ or

$$\frac{1}{4} = 0.25$$

and remember that $0.25 = \frac{25}{100}$.

“And if one-fourth is point two five, then two-fourths, or one-half, must be twice that, or point five zero, and three-fourths must be three times bigger, or point seven five:

$$\frac{2}{4} = \frac{1}{2} = 0.50 \qquad\qquad \frac{3}{4} = 0.75$$

Do you see that, Kayla?”

“Yes, that makes sense to me,” I said. “And four-fourths must be four times one-fourth, or four times point two five, which is just one, and that’s what it has to be!

$$\frac{4}{4} = 4 \times \frac{1}{4} = 4 \times 0.25 = 1$$

It all fits together!”

“Good, Kayla! Now let’s see you calculate the decimal equivalent of one-eighth.”

“Okay,” I said, “I start with the division problem: $8\overline{)1.00}$

and just do long division, like this:

$$\begin{array}{r} 12 \\ 8\overline{)1.00} \\ \underline{8}\downarrow \\ 20 \\ \underline{16} \\ 4 \end{array}$$

"But wait! There's nothing left to bring down."

Ms. Gibbs said, "All you have to do is add another zero to the dividend. When zeros come after the decimal point they don't change the value of the number. **1.00** is the same as **1.000**, remember?"

"Okay; now my long division looks like this:

$$\begin{array}{r} .125 \\ 8\overline{)1.000} \\ \underline{8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \end{array}$$

and I remembered to put the decimal point in the quotient! So the fraction, one-eighth, is the same as the decimal, point one two five:

$$\frac{1}{8} = 0.125$$

"And because the **5** is in the 'Thousandths' place, this is the same as one hundred and twenty-five thousandths:

$$0.125 = \frac{125}{1000}$$

"That right, Kayla!" said Ms. Gibbs. "But, Kayla, because you are dividing by a single digit, you could have used short division instead of long division. Let me see if you remember how to do that."

“Oh, I think I remember. First I write the division problem with spaces in the dividend:

$$8\overline{)1.\,0\;0\;0}$$

And I remembered to put the decimal point in the quotient right above the decimal point in the dividend!”

“That’s right, Kayla, good for you!”

“Now $\mathbf{8}$ goes into $\mathbf{10}$ once with $\mathbf{2}$ left over; $\mathbf{8}$ goes into $\mathbf{20}$ twice with $\mathbf{4}$ left over, and $\mathbf{8}$ goes into $\mathbf{40}$ exactly $\mathbf{5}$ times:

$$\begin{array}{r} .\,1\;\;2\;\;5 \\ 8\overline{)1.\,0^{2}0^{4}0} \end{array}$$

And I get the same answer as when I used long division, except short division is much faster.”

“That’s true, Kayla,” said Ms. Gibbs, “but it doesn’t matter which sort of division you use. You should use whichever you feel more comfortable with, because either way gives the correct answer.

“There is another thing about decimals you should be aware of, Kayla, and that is the fact that sometimes decimals can repeat themselves forever.”

“Forever?! That doesn’t sound right, Ms. Gibbs. I never heard of anything that goes on forever.”

Ms. Gibbs only said, “Well, let me see you calculate the decimal equivalent of one-third.”

I did the division and I put down three zeros just in case I needed them.

$$3\overline{)1.000}$$

Then I started to do the division:

$$
\begin{array}{r}
.333 \\
3\overline{)1.000} \\
\underline{9} \\
10 \\
\underline{9} \\
10 \\
\underline{9}
\end{array}
$$

Now I could see what Ms. Gibbs meant. Every time I multiply, subtract and bring down, the same thing happens! If I keep adding zeros to the dividend it really could go on forever!

"Do you see, Kayla?" Ms. Gibbs asked. This kind of quotient is called a 'repeating decimal' and there is a special way of writing it. A bar is placed over the part that repeats, so we would write:

$$\mathbf{\frac{1}{3} = 0.\overline{3}}$$

"The bar just means we keep repeating threes forever.

"Sometimes there is more than one digit in the part that repeats. For example,

$$\mathbf{\frac{1}{11} = 0.\overline{09}}$$

"Which means one-eleventh equals **0.0909090909**… forever. And this one:

$$\mathbf{\frac{1}{7} = 0.\overline{142857}}$$

"Which means one-seventh equals 0.142857142857142857… forever."

"That's sorta weird!" I said. "But if I follow the algorithm I can see how it happens."

“I’m so glad you understand, Kayla,” said Ms. Gibbs.

“And now that we’re learning about decimals, we can review what we learned about multiplying and dividing by powers of ten and extend it to numbers containing a decimal point.

“Actually, all numbers have a decimal point; whole numbers have one after the last digit, but it is customary not to write it down. When we write the number, 5, we don’t usually write ‘$5.$’ but we have to know the decimal point is there when applying the extended rules for multiplying and dividing by ten:

To multiply by powers of ten, move the decimal point to the right; one space for each zero in the multiplier. If there are no digits to the left of the new decimal point, add zeros as needed to reach the decimal point.

“The old way of multiplying 5 times 10 was to add a zero to the five. But when using this new rule, we move the decimal point, which we know is right after the 5, one place to the right, and add a zero. The result is exactly the same, of course, but the new rule covers all cases while the old rule just covered whole numbers.

“If we multiply 0.25 times $1{,}000$, we move the decimal point three places to the right and we have to add one zero:

$$0.25 \times 1{,}000 = 25\ . = 250$$

“We would get the same answer using regular multiplication, but that would require a lot of unnecessary work when all we have to do is move the decimal point to the right the correct number of places.

“Here’s the rule for division:”

To divide by powers of ten, move the decimal point to the left; one space for each zero in the divisor. Add zeros as needed

if there is empty space between the decimal point and the left-most digit.

"The old way of dividing by powers of ten worked only if there were zeros in the dividend. We just counted the number of zeros in the divisor, then crossed out that number of zeros in the dividend. But the new rule works for any dividends.

"For example, $\mathbf{327}$ divided by $\mathbf{10{,}000}$ equals $\mathbf{0.0327}$:

$$\frac{\mathbf{327}}{\mathbf{10{,}000}} = \mathbf{0.\ \ 327} = \mathbf{0.0327}$$

"We moved the decimal point in $\mathbf{327}$ four places to the left because there are four zeros in the divisor, and added one zero to fill in the one space between the decimal point and the $\mathbf{3}$.

"We could get the same answer using long division, like this:

```
            .0327
      ___________
10000)327.0000
      300 00↓ |
        27 000|
        20 000↓
         7 0000
         7 0000
```

but that would be doing a lot of unnecessary work when all we need to do is move the decimal point the correct number of spaces to the left.

"Would you like to try one, Kayla? Let me see you multiply forty-seven thousandths by one hundred."

"Okay, I can do this. Forty-seven thousandths means the last number, which is seven, is in the thousandths place, so it is written $\mathbf{0.047}$. And one hundred has two zeros in it, so I move the decimal point two places to the right:"

$$0.047 \times 100 = 4.7$$

"Very good, Kayla. Using the new algorithm shows you are getting more math savvy. I often see high school students multiplying it all out. That shows they don't understand about powers of ten.

"Now divide **26.3** by **100,000**."

Reader, can you figure this out before you read how Kayla does it?

I said, "One hundred thousand has…let me see…two zeros for the hundred and three zeros for the thousand makes five zeros, so I have to move the decimal point in **26.3** five places to the left. And when I do that, there are three zeros to put between the decimal point and the two":

$$\frac{26.3}{100{,}000} = 0.000263$$

"Very good, Kayla! Now there's just one more thing about decimals, and then we'll talk about per cents. That one thing is dividing by a decimal.

"Kayla, you just learned that when there is a decimal point in the dividend, you must put a decimal point in the quotient exactly above that decimal point. However, when there is a decimal point in the *divisor* there is something you must do first.

"**If there is a decimal point in the divisor, we must move it to the right until the divisor is a whole number.**"

"Huh? Can we just do that?" I asked.

"Kayla, your math savvy tells you we *can't* just do that! We have to move the decimal point in the dividend exactly the same number of spaces to the right as we moved the decimal point in the divisor.

“That sounds confusing, so let me just show you an example.

“How would you divide one by point five?”

$$.5\overline{)1}$$

“Well, you said the divisor must always be a whole number, so I have to make the **.5** into a whole number by moving its decimal point one space to the right. And so we must also move the decimal point in the dividend one space to the right. But there’s nothing between the one and the decimal point, so I’ll put a zero there and put a decimal point in the quotient exactly above the one in the dividend:

$$\begin{array}{r} 2\,. \\ 5\overline{)10.} \end{array}$$

Reader, if you don’t remember the names for the division terms, you can review them in Book 7, page 18.

“I know that **5** goes into **10** twice, and so the answer is **2**. There are two ‘point fives’ in one.” I looked up at Ms. Gibbs for her nod.

Ms. Gibbs nodded and said, “That’s right, Kayla. And you saw that point five is the same as one-half, so obviously there are two one-halves in one.

“Let’s do another example. This time you don’t have to do the division. Just show me where the decimal points in the dividend and the divisor should go”:

$$24.76\overline{)143.98}$$

“That’s easy!” I said. To make the divisor a whole number, we move the decimal point two places to the right. But we must also move the decimal point in the dividend two places to the right”:

$$\mathbf{2476\overline{)14398}}.$$

“That’s right, Kayla. Though not written, there are decimal points after the **6** and after the **8**. The decimal point in the quotient must be exactly above the decimal point after the **8**, just as you have written it.

“Now I’d like to talk with you about per cents. Compared to decimals, per cents will be much easier to understand, Kayla,” said Ms. Gibbs.

Chapter 5

Per cents

"Remember that when we studied division, we found that 'per' means 'for each'. And 'cent' is often used to mean 'one hundred.' (You know that there are one hundred cents in a dollar.) It makes sense, then, that 'per cent' just means 'for each one hundred'.

"For example, suppose you take a test that has twenty questions on it and you get sixteen of them right. You got '$\mathbf{16}$ per $\mathbf{20}$'; what per *cent* is that? In other words, $\mathbf{16}$ per $\mathbf{20}$ is how many per $\mathbf{100}$?

"We want to know how many out of $\mathbf{100}$ is the same as $\mathbf{16}$ out of $\mathbf{20}$. You know 'the same as' means 'equal to' in math, so we can write:

$$\frac{?}{\mathbf{100}} = \frac{\mathbf{16}}{\mathbf{20}}$$

"And if we multiply both sides by $\mathbf{100}$, we have:

$$\cancel{\mathbf{100}} \times \frac{?}{\cancel{\mathbf{100}}} = \mathbf{100} \times \frac{\mathbf{16}}{\mathbf{20}}$$

$$\mathbf{?} = \mathbf{100} \times \frac{\mathbf{16}}{\mathbf{20}}$$

"Now we can divide the top and bottom by $\mathbf{10}$ to get:

$$\mathbf{?} = \frac{\mathbf{16}}{\mathbf{2}} \times \mathbf{10}$$

"which is just $\mathbf{8} \times \mathbf{10} = \mathbf{80}$, so $\mathbf{16}$ correct out of $\mathbf{20}$ is a score of $\mathbf{80}$%.

"Another way of solving an equation like this is to use **cross-multiplication**.

“Hmm…,” I thought, “It seems in math there are always more than one way of doing anything,” but I didn’t say anything to Ms. Gibbs.

She continued, “Whenever two fractions are equal, the top left times the bottom right equals the bottom left times the top right. In this case, we can visualize this using arrows:

$$\frac{?}{100} = \frac{\mathbf{16}}{\mathbf{20}}$$

writing these products we have:

$$\mathbf{?} \times \mathbf{20} = \mathbf{100} \times \mathbf{16}$$

and it is easy to see that the question mark must be equal to **80.**

“We can make a simple algorithm for calculating per cents based on the third equation we wrote down:

$$\mathbf{?} = \mathbf{100} \times \frac{\mathbf{16}}{\mathbf{20}}$$

“Per cent is calculated by dividing the part (**16** in this example) by the whole (**20** in this example), then multiplying by **100**. But we now know that multiplying by **100** is the same as moving the decimal point two places to the right, so we can rephrase this algorithm as follows:

Per cent is calculated by dividing the part by the whole, then moving the decimal point two places to the right.

“Using this algorithm, we calculate the per cent you scored on your test simply by dividing **16** by **20** to get **0.80**, and moving the decimal point two places to the right to get **80%**.”

“This does seem a lot simpler than decimals,” I said. “It seems like a per cent is just a fraction changed into a decimal, with the decimal point shifted two spaces.”

“That summarizes how to calculate per cents very well, Kayla. I think someday you could be a fine math teacher!” Ms. Gibbs said.

“When the part is equal to the whole, the per cent equals **100**. For example, had you gotten every answer correct on your test, you would have scored **20** out of **20**:

$$\frac{\mathbf{20}}{\mathbf{20}} = \mathbf{1.00}$$

“And when we move the decimal point two places to the right, we get **100**%, which you know means you got all the answers right.

“Though not very common, per cents can be greater than **100**%. You could never get more than **100**% on a test, but sometimes quantities can increase by more than **100**%. We won’t have to concern ourselves with those special cases for now.

“The last topic for today is how to calculate an amount given the per cent and a total number.

“A practical example of this is the amount of tax one has to pay given the total cost and the per cent of the tax rate. In many states, the sales tax is seven per cent. If the total cost of an item were $**2.00**, the tax would be **7**% of this.

“You already know that **7**% means **7** per hundred, which may be written as a fraction:

$$\mathbf{7}\% = \frac{\mathbf{7}}{\mathbf{100}}$$

“**7**% *of* $**2.00** tells us to *multiply* this fraction times $**2.00,** so the amount of tax for this purchase would be:

$$\frac{\mathbf{7}}{\mathbf{100}} \times \$\mathbf{2.00} = \$\mathbf{0.14}$$

And you know that $**0.14** is the same as **14¢.**

“You can see that we can create an algorithm for this kind of calculation:

To obtain an amount from a total and a per cent, multiply the per cent times the total and move the decimal point two places to the left.

“You can see an inverse relationship here: when we want to find a per cent, we move the decimal point two places to the right, and when we use per cent in a calculation, we move the decimal point two places to the left.

“As we just discussed, a real-life example of this kind of calculation is the tax you pay when buying something at the store.”

“Tax?” I said, “But Ms. Gibbs, I sometimes buy bread at a store near my house and I don’t remember paying any tax.”

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“Well,” replied Ms. Gibbs, ‘in Florida, there is no tax on food you buy in the store. But if you buy food in a restaurant, or anything else in a store, you must pay the sales tax.

“Suppose the sales tax is 7%, and you buy a sweater for $12. How much would the tax on your purchase be?” Ms. Gibbs asked.

“The algorithm says to multiply the per cent times the total, and move the decimal point two places to the left. The percent is 7 and the total is $12:

$$\begin{array}{r} {}^{1} \\ \$12 \\ \underline{x7} \\ \$84 \end{array}$$

“Now I have to move the decimal point two places to the left, to get

$$\$0.84$$

"Which is the same thing as $84¢$. Wow! So if I save up \$$12$ to buy this sweater, I wouldn't have enough money. I actually need $84¢$ more, or almost \$$13$!"

"That's right, Kayla," said Ms. Gibbs, "and when someone makes a larger purchase, the tax can be quite high. Can you tell me what the sales tax would be if an item costs \$$1200$?"

I started to multiply 1200 times 7, but then I realized that this is just one hundred times bigger than the problem I just did for \$$12$. So the answer must be one hundred times bigger! And to multiply \$$0.84$ times 100, just move the decimal point two places to the right.

"The answer is \$$84$," I said, "That's really a lot of money extra just because of sales tax"!

"Good for you, Kayla," Ms. Gibbs said, "You can see that people must take the sales tax into account when they go shopping.

"Now there's one last calculation using per cents that I want to tell you about. It takes advantage of your math savvy."

"My math savvy?" I thought. That made me curious.

Ms. Gibbs continued, "When people go out to eat at a restaurant, it is customary to leave a 15% tip for the waitstaff. You know already that you must first multiply 15 times the total bill to figure out the size of the tip. But for most people this requires pencil and paper. Using your math savvy, however, you can do this calculation in your head."

"Hmmm…," I thought, "my math savvy doesn't know what to do."

Ms. Gibbs must have read my mind, because she said, "You may not see how to do this right now, but I'm sure once I show you, your math savvy will have you say, 'I get it!'

“If you think about it, you’ll see that 15% is just the sum of 10% and 5%. Calculating 10% of anything means multiplying by 10 and moving the decimal point two places to the left. You can see that the end result is just moving the decimal point in the total bill *one* place to the left. This you can do in your head for sure!

“And you know that 5% is just one-half of 10%. You already have 10%, so you just have to take one-half of that number to get 5%. Dividing by two is something you can do in your head, especially because you don’t need an *exact* answer. Then just add these two results together to get a good estimate of 15% of the bill.

“Let’s see you figure out the tip if the bill for your meal comes to $8.00.”

“OK, I can do this,” I said. “I get 10% of $8.00 by moving the decimal point one place to the left. That’s $0.80. Now 5% is one-half of this, $0.40. The sum of my 10% and my 5%, which is 15%, is $1.20.

“The tip would be $1.20, and the total bill would be $9.20. And I *do* understand why this trick works, Ms. Gibbs,” I said.

“I knew you would, Kayla,” said Ms. Gibbs.

I thought we were finally finished for today and I was almost out the door when I heard Ms. Gibbs say, “Wait, Kayla. I almost forgot. There is one more thing we need to do before we’re finished for today.”

As Ms. Gibbs said that, she took my multiplication grid from her folder and held it out for me to see.

I looked at the grid and exclaimed: “Why I’m almost done!” I knew I was almost done but it was good to see my grid with only a few blank boxes.

And I have some more numbers to put in it today. Then I'll *really* be almost done. I put the numbers I had learned this last week into the grid.

I learned them by saying them over and over again. That trick really works! I filled in the rest of my three times tables, 3 x 6 is 18, 3 x 7 is 21, and 3 x 8 is 24 and…and 6 x 3 is 18, Wait, I filled that one in already…and 6 x 8 is 48.

After I put those numbers in my grid, I wanted to count the empty boxes but Ms. Gibbs put my grid away and just said there weren't too many. And then she said, "You'll be able to finish the entire grid next week!"

I remember when I first started the grid, I thought I never could remember all those multiplication facts. One hundred and twenty-one of them! But by memorizing facts just a few at a time, I was able to do it.   I hope you're learning your multiplication facts too.

The next page shows what Kayla's Multiplication Grid looks like now.

Multiplication Grid

	1	2	3	4	5	6	7	8	9	10	11
1	1	2	3	4	5	6	7	8	9	10	11
2	2	4	6	8	10	12	14	16	18	20	22
3	3	6	9	12	15	18	21	24	27	30	33
4	4	8	12	16	20	24	28	32	36	40	44
5	5	10	15	20	25	30	35	40	45	50	55
6	6	12	18	24	30	36	42	48	54	60	66
7	7	14		28	35	42	49		63	70	77
8	8	16		32	40			64	72	80	88
9	9	18	27	36	45	54	63	72	81	90	99
10	10	20	30	40	50	60	70	80	90	100	110
11	11	22	33	44	55	66	77	88	99	110	121

Chapter 6

Reviewing what I learned

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Today I learned about decimals. I want to tell you something neat I learned about them right away - just in case I forget. It's this: You can put a decimal point - that's just like a period - after *any* whole number and then add as many zeros as you'd like, even a million, and it won't matter - the value of the number doesn't change. Really, it doesn't!

And you can do the same thing after the *last number* after the decimal point. Wait, I'll show you what I mean: **0.4** and **0.40000** mean *exactly* the same thing. Really, they do! I found that interesting and thought you would too.

Well, anyway, let's get serious because decimals are serious. You have to learn them if you want to be good at math or if you just want a good grade in math.

If there's just one number to the right of the decimal point, the decimal is "tenths"; for example, **2.8** would be "two and eight tenths."

If there are two numbers to the right of the decimal point, the decimal is "hundredths"; for example, **5.34** would be "five and thirty-four hundredths."

If there are three numbers to the right of the decimal point, the decimal is "thousandths"; for example, **6.301** would be "six and three hundred and one thousandths."

You would only rarely see more than three numbers to the right of the decimal point, so that's really all you have to remember - for now.

And another reason you have to learn about decimals is to be a good shopper. You see, decimals are just like money – well they could be.

Money has a period dividing the dollars from the cents - only it's not called a period, it's called a decimal point. But it looks the same. Here, I'll show you! First money and then just a plain decimal:

$5.34 or **5.34**

When I read the first, I would say: "Five dollars and thirty-four cents." The second one I would read, "Five and thirty-four hundredths." That's how you say it if it's just a decimal number.

Just remember, a decimal describes something less than one - like a fraction. Now I'm talking about the number that's to the right of the decimal point. Any number to the left of the decimal point is just a plain old whole number. Remember that! For example, in the number above, **5.34**, the "**5**" is a plain old whole number and the "**.34**" is the decimal and together they read "**5** and **34** hundredths." Get it?

The reason we know it's **34** *hundredths* is that the *first* space after the decimal point is *tenths*; the *second* one is *hundredths*; the *third* one is *thousandths*, and so on. So if the number was just **5.3**, it would be five and three tenths. But because the last number is in the *second* space after the decimal point, we know the decimal is a *hundredth* so it's **34** hundredths. Got that?

Here's a place value table, like the one Ms. Gibbs showed me, but this one includes decimals:

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
		5	**.3**	**4**	

The decimals start right after the Ones place. I put a decimal point in front of the **3** to make it clear. And just like for whole numbers, each place going right is one-tenth of the place before it.

You can see what **5.34** looks like in the Place Value Chart. There are **5** Ones, **3** Tenths and **4** Hundredths. Remember that when there are two numbers to the right of the decimal point, the decimal is read as "hundredths" so the number in the table is "five and thirty-four hundredths."

Because each column is ten times bigger than the one on its right, each Tenth equals ten Hundredths, so **3** Tenths is the same as **30** Hundredths. That means **3** Tenths, which is **30** Hundredths, plus **4** Hundredths equals **34** Hundredths.)

In the practice problems, I have a lot of decimal numbers so you can practice reading them. It's a little tricky but if you practice, you won't be tricked.

And when we say, "thirty-four hundredths," this reminds us that **0.34** is the same thing as **34** divided by **100**:

$$\mathbf{0.34} = \frac{\mathbf{34}}{\mathbf{100}}$$

So really decimals and fractions are just different ways of saying the same thing.

You might want to read this whole section again, because, well, because it's pretty tricky so don't be tricked by it!

Let's look at another decimal in this Place Value Table: **27.013**

Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
	2	**7**	**.0**	**1**	**3**

Here we have **2** Tens, which is twenty, plus **7** Ones, which make twenty-seven – those are just regular numbers. Got that? Now after the decimal point we have three numbers, so we know they are thousandths: we read this number as "twenty-seven and thirteen thousandths."

It's important to remember that when we say "thirteen thousandths" we can think of this as a fraction:

$$\mathbf{0.013} = \frac{\mathbf{13}}{\mathbf{1000}}$$

So you can see how easy it is to express a decimal as a fraction. Well, it will be easy after a little practice. Just remember when we change a decimal into a fraction the denominator is always a power of ten: **10**, **100**, **1000** and so forth.

Converting a fraction to a decimal is not so easy. You have to do a division problem. Watch me do one. Suppose you want to know the decimal equivalent of the fraction, three-eighths:

$$\frac{3}{8} = 8\overline{)3.000}$$

This is good practice for you to work on short division! Remember how we left spaces in the dividend, like this?:

$$\begin{array}{r} 0.\,3\;7\;5 \\ 8\overline{)3.\,0^{6}0^{4}0} \end{array}$$

And **8** goes into **30, 3** times with a remainder of **6**, **8** goes into **60, 7** times with a remainder of **4**, and **8** goes into **40** exactly **5** times. Remember?

So we now know that

$$\frac{3}{8} = 0.375$$

And don't forget that **0.375** = $\frac{375}{1000}$ **,** so $\frac{3}{8}$ and $\frac{375}{1000}$ must be equivalent fractions.

Reader, if you want to review "Equivalent Fractions" see Book 5.

Please try to remember this about $\frac{3}{8}$ because I'm going to show you how to convert it into a per cent. That will be after we're all finished talking about decimals!

Adding and subtracting decimals

The rules are the same for adding and subtracting decimals so I'll explain them together. When you add or subtract decimals, it's really important to <u>keep the decimal points right under each other.</u> I underlined it for you because it's so important. And I'll do it for you, too: **3.52** and **0.4** look like this when you're adding or subtracting them:

$$\begin{array}{r} 3.52 \\ +0.4 \\ \hline 3.92 \end{array} \qquad \begin{array}{r} 3.52 \\ -0.4 \\ \hline 3.12 \end{array}$$

Oh, I put that extra zero before the **.4** just so you'll know for sure it's a decimal.

So the most important thing to remember when adding and subtracting fractions is to put the decimal points right under each other and then you can just add or subtract. And just make sure you put the decimal point in the answer right under the decimal points in the numbers you're adding or subtracting. That's important. Got it?

Multiplying decimals

Multiplying and dividing decimals are a lot different from adding and subtracting them and they are different from each other. So I need to explain them separately. Let me explain multiplying first.

When multiplying, you don't have to line up the decimal points. Just multiply the numbers in the usual way. But to find out where the decimal point goes in your answer, you count the total number of digits that lie *to the right* of the decimal points in *both numbers* and then put the decimal point in your answer that many spaces from the right.

As always, as Ms. Gibbs says, an example makes it much easier to understand. Suppose I want to multiply **1.52** times **0.2**. I'd do It like this:

$$\begin{array}{r} \mathbf{1.52} \\ \underline{\times\mathbf{0.2}} \\ \mathbf{.304} \end{array}$$

See what I did? I multiplied, just like they're regular numbers. Then I counted the digits - all together - to the right of the decimal points; there are *three*: the **5** and **2** in the top and the **2** in the multiplier. And then, for the answer, I put a decimal point *three* spaces from the right in the answer. Got it?

Dividing decimals

Dividing decimals is different. I'll just show you what the difference is by comparing division by a whole number with division by a decimal. Let's first divide **2.42** by **2**, and then we'll divide **2.42** by **0.2**.

When you divide by a plain whole number, you just have to remember about where the decimal point goes in the quotient:

$$\begin{array}{r} \mathbf{1.21} \\ \mathbf{2}\overline{\smash{)}\mathbf{2.42}} \end{array}$$

See, I just divided and I made sure the decimal point in the quotient is exactly *above* the decimal point in the dividend. Got it?

What if you want to divide by a decimal; **0.2** instead of **2**? Well you can't do that! You just can't. Really! That's what Ms. Gibbs said.

You have to first change the divisor into a whole number before you can divide by it. But the way you do that is pretty tricky, so pay attention!

$$0.2\overline{)2.42}$$

In this problem, in the divisor, **0.2*,*** there is one number to the right of the decimal point, and that makes it a decimal. Since I can't divide by a decimal - that's the rule - I'll just need to make it a whole number. Really, that's what I need to do!

I can do that by moving the decimal point in the divisor one place to the right to make **0.2** a plain old **2.** But now, in the dividend, I have to move the decimal point one place to the right too, or else I'll get the wrong answer. Pretty tricky!

Now, I didn't make this up, that's how you do it. As I said, "it's the rule." So now my division problem looks like this:

$$2\overline{)24.2}$$

And then I just divide. Wait, I'll do it for you:

$$\begin{array}{r} 12.1 \\ 2\overline{)24.2} \end{array}$$

See what I did? I just had to make sure the decimal point in the quotient is *right above* the decimal point in the dividend. That's it!

It's really important that you understand decimals because they're so tricky and you might be tricked by them. If you understand them really well, they won't trick you.

Oh, one more thing, and this is very important! When multiplying decimals by powers of **10**, make sure to use that short cut –

remember that one?   You don't need to multiply the numbers out, you can just move the decimal point over one or more spaces to the right like this:

5.3 x **10** = **53**

See what I did? When I multiplied by ten, I just moved the decimal point one place over to the right.

But what if you want to multiply by **100** or **1000** or more and there aren't any more spaces in the **5.3**? Well you just add zeros in the answers like this:

5.3 x **100** = **530**
5.3 x **1000** = **5300**

And even like this:

5.3 x **10,000** = **53,000**

Go ahead and multiply them out. Just pretend you don't know that trick - oh, I mean that short cut - just to prove it to yourself. Then use the short cut. That's what I did.

And of course the only difference when you're *dividing* by powers of 10 is that you move the decimal point to the *left* instead of to the *right*.

So now you know how to add, subtract, multiply and divide decimals. Per cents are not so hard if you know your decimals, so if you understand them, per cents will be easy for you.

Per cents

Remember when we learned about that word "per"? "Per" just means "for each." Well, "per cent" just means "for each *one hundred*." For example, **35**% means **35** for each **100**. If we had **100** children and **35**% of them were boys, there would be **35** boys. If we had **200** children and **35**% of them were boys, there would be **70** boys; **35** for *each* of the two **100**'s.

If you take a test with **10** questions on it and get **9** correct, the per cent you got right is **90**% because **9** out of **10** is the same as **90** out of **100**, and per cent is per **100**.

Since you know all about decimals, you already know a lot about per cents. To change a decimal into a percent you just move the decimal point two spaces to the right and add a percent sign like this:

$$.56 = 56\%.$$

Really, it's that simple! And to change a percent into a decimal, move the decimal point two spaces to the left and drop the % sign.

Finding a per cent is pretty easy, because it's really just a different way of thinking about a fraction. Remember, a fraction is just the part divided by the whole.

Reader, if you want to review how we find fractions, that's in Book 1.

We convert a fraction into a decimal by division, and then just move the decimal point two places to the right to get the per cent.

Remember that example I did of converting the fraction, $\frac{3}{8}$, to the decimal, **0.375,** by division? To get the per cent we just move the decimal point two places to the right and add the % sign: **37.5%**. Three is **37.5%** of eight. It's that simple! Here, I'll do it for you again, but with a different fraction.

Watch me convert the fraction, $\frac{2}{5}$, into a per cent:

First I convert the fraction to a decimal by dividing:

$$5\overline{)2.00}\quad .40$$

Then all I have to do is move the decimal point two places to the right and add the % sign:

$$\frac{2}{5} = \mathbf{0.40} \longrightarrow \mathbf{40\%}$$

There is another way to change a fraction into a per cent. Let's pretend you took a test with **20** questions and got **10** right. The fraction you got right is just **10** over **20**. You rewrite the fraction as an equivalent fraction with the unknown per cent in the numerator and **100** in the denominator:

$$\frac{10}{20} = \frac{?\%}{100}$$

Pay attention because I'm going to show you a good trick. First you multiply both sides by **100** - just do it - and you'll see why.

$$100 \times \frac{10}{20} = \frac{?\%}{100} \times 100$$

On the right-hand side, $100/100 = 1$, so all that's left over on the right hand side is "?%" Well, now you know the reason why you do it! That must equal the left-hand side, which is 50.
Your grade would be 50%. That's not good, but remember, we're just pretending!

Here's another problem. Suppose you studied your per cents and you go to a store and the sign says "40% off the original price," and suppose the original price is \$$1.00$. Here's what you do: You multiply \$$1.00$ x 40, and move the decimal point two places to the left, to get \$$0.40$. That's the same as 40¢, so the cost would be \$$1.00$ minus 40¢ or 60¢.

Here's another problem: Suppose you go to a restaurant and you get a bill for 10 dollars and you want to figure out the tip. You decide to give 15% of the cost of the meal. Multiply \$$10$ by 15 and move the decimal point two places to the left:

$$\$10 \times 15 = 150 \rightarrow \$1.50$$

Suppose you want to buy a book. The sign over the book says \$$5.00$ but then there is another sign which says you get 25% off the regular price. Here's how you do it:

$$\$5.00 \times 25 = 125 \longrightarrow \$1.25$$

The answer you get is how much money you get *off* the regular price. Well, to learn how much that book costs, you subtract how much you get off from the regular price. I'll do it for you:

$$\begin{array}{r} \$5.00 \\ \underline{-\$1.25} \\ \$3.75 \end{array}$$

The book will only cost $3.75. That's sound like a good deal. I'd buy the book if I liked it and if I had the $3.75.

You can see how important it is to know about per cents and how important it is to know how to multiply and divide. So keep learning this stuff and you'll be a good shopper!

Practice problems

1. Read the following numbers using tenths, hundredths and thousandths for the decimal parts:

a) **0.7** b) **0.07** c) **0.007**

d) **0.15** e) **0.355** f) **0.553**

g) **1.75** h) **25.25** i) **64.951**

2. Powers of ten practice. Watch the signs!

a) $\mathbf{31} \times \mathbf{100} =$ ___________ b) $\mathbf{3.1} \times \mathbf{1000} =$ __________

c) $\mathbf{124} \div \mathbf{10} =$ ___________ d) $\mathbf{0.065} \times \mathbf{100} =$ ___________

e) $\mathbf{983} \div \mathbf{10{,}000} =$ ___________ f) $\mathbf{10} \times \mathbf{1000} \div \mathbf{10} =$ ___________

3. Give the decimal equivalents of the following fractions:

a) $\frac{\mathbf{3}}{\mathbf{10}} =$ ___________ b) $\frac{\mathbf{27}}{\mathbf{10}} =$ ___________

c) $\frac{\mathbf{73}}{\mathbf{1000}} =$ ___________ d) $\frac{\mathbf{8}}{\mathbf{100}} =$ ___________

4. Give the decimal equivalents of the following fractions: If your answer is a repeating decimal make sure you use a bar**:**

a) $\frac{\mathbf{2}}{\mathbf{5}} =$ _____________ b) $\frac{\mathbf{2}}{\mathbf{3}} =$ _____________

c) $\frac{\mathbf{3}}{\mathbf{8}} =$ _____________ d) $\frac{\mathbf{8}}{\mathbf{11}} =$ _____________

e) $\frac{\mathbf{4}}{\mathbf{9}} =$ _____________ f) $\frac{\mathbf{5}}{\mathbf{8}} =$ _____________

5. Perform the following division problems:

a) $25\overline{)10}$

b) $3.5\overline{)10.5}$

c) $0.004\overline{)20.8}$

d) $4.75\overline{)1.21125}$

6. Give the per cent that is the same as the following fractions. Again, use bars for repeating decimals:

a) $\frac{1}{2}$ = ______%

b) $\frac{1}{4}$ = ______%

c) $\frac{3}{5}$ = ______%

d) $\frac{2}{3}$ = ______%

e) $\frac{3}{8}$ = ______%

f) $\frac{5}{6}$ = ______%

7. If Hilda got 18 answers correct out of 20 questions, what was her score expressed as a per cent?

__________%

8. **750** students had to take a reading test. **57** students failed the test.

a) What per cent of the students failed the test?

__________%

b) What per cent of the students passed the test?

__________%

9. **15** minutes is what per cent of one hour? __________%

10. One mile is **5,280** feet. Which is greater, **600** feet or **10**% of one mile?

____

11. You have a coupon that gives you either **20**% or $**5** off the list price. You want to buy an item that regularly costs $**23**.

a) Should you choose **20**% off or $**5** off, to save more money?

____

b) You must pay a **7**% sales tax on the cost of the item *before* the discount. How much do you pay in sales tax for this item?

12. You and some friends go out for pizza, and the bill comes to $30.00. You want to leave a 15% tip. How much should you leave on the table for the wait staff?

13. Consider again the coupon that gives you either 20% off the price or a $5 discount. What is the price that makes these two discounts exactly the same?

14. The population of Sarasota, FL, is about 56,600. About 17% of the population is under 18 years old, and about 30% of the population is over 65 years old.

a) About how many people are under 18 years old in Sarasota?

b) About how many people are over 65 years old in Sarasota?

c) About what per cent of people in Sarasota are between **18** and **65** years old?

____

d) About how many people is this?

____

e) What is the sum of your answers to a), b) and d)?

____

Something extra

Maps! Maps are a good way to learn about the place we live in or places we just might travel to someday. Sometimes maps show a small area, like a neighborhood and sometimes they show the whole world. Those that show the whole world are called, guess what! World maps! And do you know what else? Sometimes a world maps is a globe. That's because the world is round, just like a globe.

The most basic things you need to know when reading a map are the directions. North is always at the top while South is always at the bottom. West is to your left and East Is to your right. When the earth rotates it looks like the sun comes up over the horizon, travels across the sky, and then disappears over the other horizon. (The "horizon" is the place where the sky meets the land.) You might have heard the saying, "The sun rises in the East and sets in the West." And the farther West you are, the later you see the sun rise.

Florida, the state that I live in, is in the South and in the East. So in a map of the United States, it's way to the right and near the bottom. Do you know what state you live in? And where it is on the map? Ask someone who knows, and then you'll know.

It's warm here in Florida almost all year round – that's because it's in the southern part of the United States. In the summer it gets really hot but in the winter it doesn't get really cold. In fact, most days, even during the winter, I wear shorts.

If you live in the North, you couldn't do that, I bet. Oh, maybe in the summer you can wear shorts. but in the winter, you need to wear a warm coat and hat and sometime boots. In places where it's real cold, it snows. I never, ever saw snow except, of course, in pictures.

Answers

1. a) seven tenths b) seven hundredths c) seven thousandths

   d) fifteen hundredths e) three hundred fifty-five thousandths

   f) five hundred fifty-three thousandths

   g) one and seventy-five hundredths

   h) twenty-five and twenty-five hundredths

   i) sixty-four and nine hundred fifty-one thousandths

2. a) $\mathbf{3{,}100}$ b) $\mathbf{3{,}100}$ c) $\mathbf{12.4}$

   d) $\mathbf{6.5}$ e) $\mathbf{0.0983}$ f) $\mathbf{1{,}000}$

3. a) $\mathbf{0.3}$ b) $\mathbf{2.7}$ c) $\mathbf{0.073}$ d) $\mathbf{0.08}$

4. a) $\mathbf{0.4}$ b) $\mathbf{0.\overline{6}}$ c) $\mathbf{0.375}$ d) $\mathbf{0.\overline{72}}$ e) $\mathbf{0.\overline{4}}$ f) $\mathbf{0.625}$

5. a) $\mathbf{0.4}$ b) $\mathbf{3}$ c) $\mathbf{5{,}200}$ d) $\mathbf{0.255}$

6. a) $\mathbf{50}\%$ b) $\mathbf{25}\%$ c) $\mathbf{60}\%$

   d) $\mathbf{66.\overline{6}}\ \%$ e) $\mathbf{37.5}\%$ f) $\mathbf{83.\overline{3}}\%$

7. **90**%

8. a) **7.6**% b) **92.4**%

9. **25**%

10. **600** feet

11. a) $**5** b) $**1.61**

12. $**4.50**

13. $**25**

14. a) **9,600** b) **17,000** c) **53%** d) **30,000** e) **56,600**

About tutoring

Tutoring isn't just about helping someone learn one particular subject. It's an opportunity for the tutor to share something that s/he knows. While Ms. Gibbs is Kayla's math tutor, she shared a little about the time she visited El Salvador. She also taught Kayla about maps. In turn Kayla also shared with Ms. Gibbs where she lives and where she goes swimming (on the map). Tutoring is just sharing information.

There are so many things to learn about the world we live in. I hope you find it fun to share some of what you know. The more we share, the more we all know. And if we all know a lot, the more likely we are to try to make the world a better place for all of us to experience.

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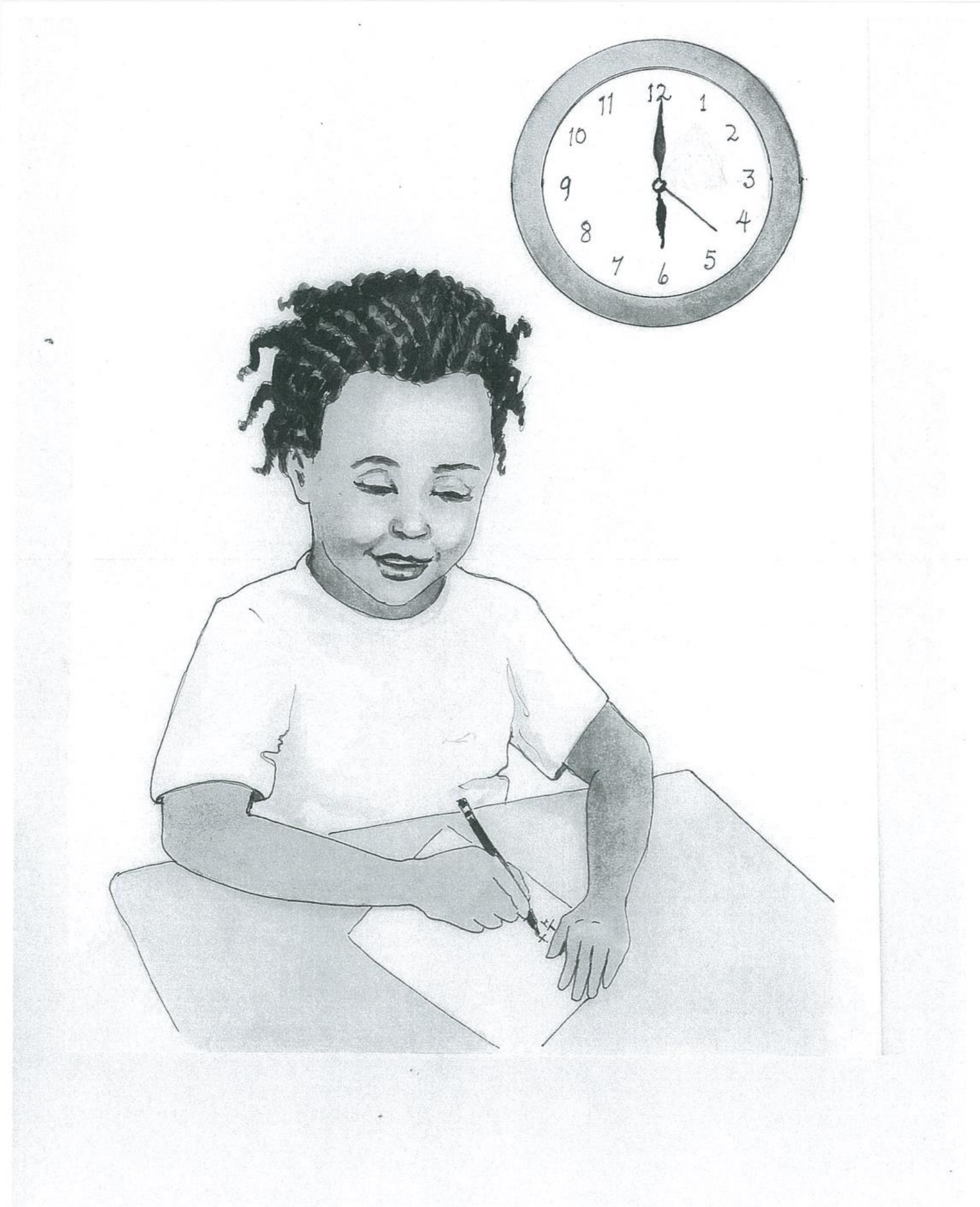
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