Grand Challenge TWO – A Statistical Analysis

By Ed Meyer

Grand Challenge TWO was:

Six standard dice are rolled. What is the probability that exactly four of the possible numbers appear? Note that a gambler in the 1700s knew that the result was very close to 50%, but he nor any mathematician he asked could calculate it. Can you?

In the previous five issues we focused on getting the answer. Here we are going to investigate the statistics of the results of a large number of rolls of six dice, which conclusions can be drawn, and the level of confidence we have in those conclusions.

We have the answer to the problem, and we can use it to determine the expected deviation of the results of a large number of rolls.

Previously, we determined that the probability that a roll of six dice would have exactly four different numbers is

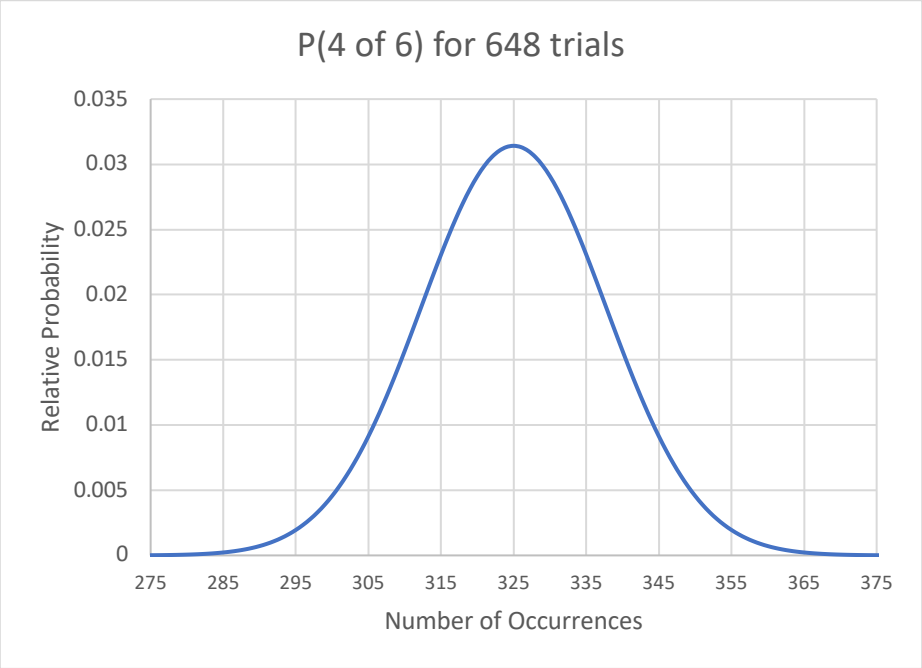
$$P = \frac{325}{648}$$

This means that if we rolled six die 648 times, we would expect that the 325 of the rolls would have exactly four different numbers. Of course, when you actually do this the results will be scattered considerably, but you would expect the average of be around 325. In one trial, you might get 320 rolls in which there were exactly four different numbers. In another, you might get 332 and in another you might get 315. It is just the nature of randomness and probabilities.

We can calculate the standard deviation of the results of 648 rolls of six dice with the formula

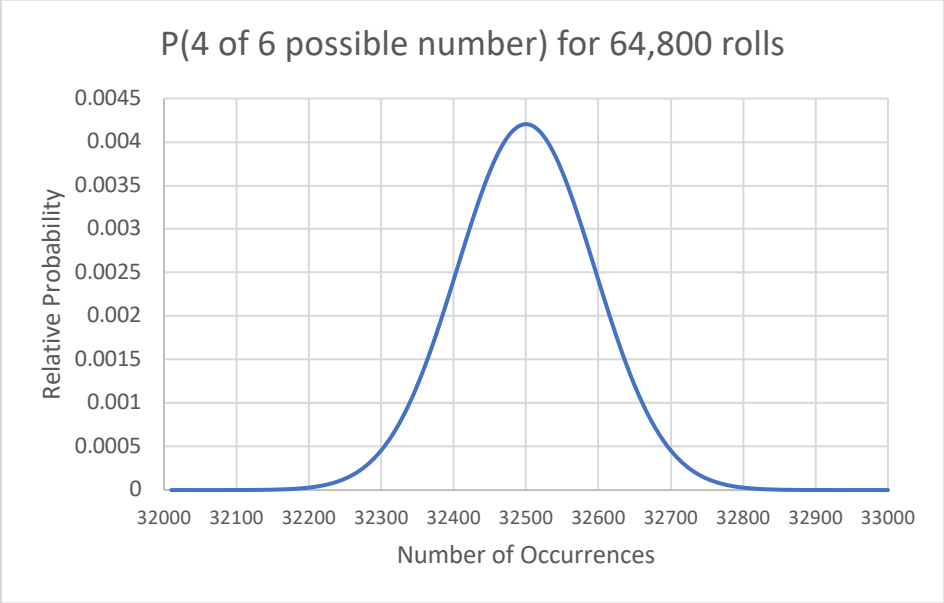
$$\sigma = \sqrt{npq} = \sqrt{648 \times \frac{325}{648} \times \frac{323}{648}}$$

where σ is the standard deviation, n is the number of trials, p is the probability of getting exactly four of the six numbers and q is the probability of not getting exactly four of the six numbers. The result is about 12.7. Let's plot the theoretical probability distribution function of 648 trials and try to understand what it means.



When six dice are rolled 648 times, the expected value of the number of rolls that contain exactly four of six different number is 325. This is the peak of the distribution. However, 648 rolls are not nearly enough to conclude that the actual expected value is 325, not 324, as 324 is just a tiny bit to the left of the peak. To experimentally make the conclusion that the probability of getting four of six numbers with a roll of six dice is not 50%, a LOT more rolls are needed.

Let's look at the continuous probability distribution function for 64,800 rolls. Is this enough to determine that the probability of getting exactly four of the six possible numbers is not 50%?



With 64,800 rolls, the expected number of times that there will be exactly four of the six numbers is 32,500. We know that because we did the calculation. If the probability was 50%, we would expect that there would be four of six numbers 32,400 times.

A look at the probability distribution above reveals that there is still a significant probability of getting 32,400 or fewer rolls in which exactly 4 of 6 numbers appear. In fact, if you roll six dice 64,800 times there is about a 23% chance that you will get fewer than 32,400 rolls in which four of the six possibilities appear.

Indeed, 64,800 rolls is not enough to conclude that the probability of exactly four of six numbers appearing when six dice are rolled is 50% because there is a 23% chance that you will get four of six possible numbers less than 50% of the time.

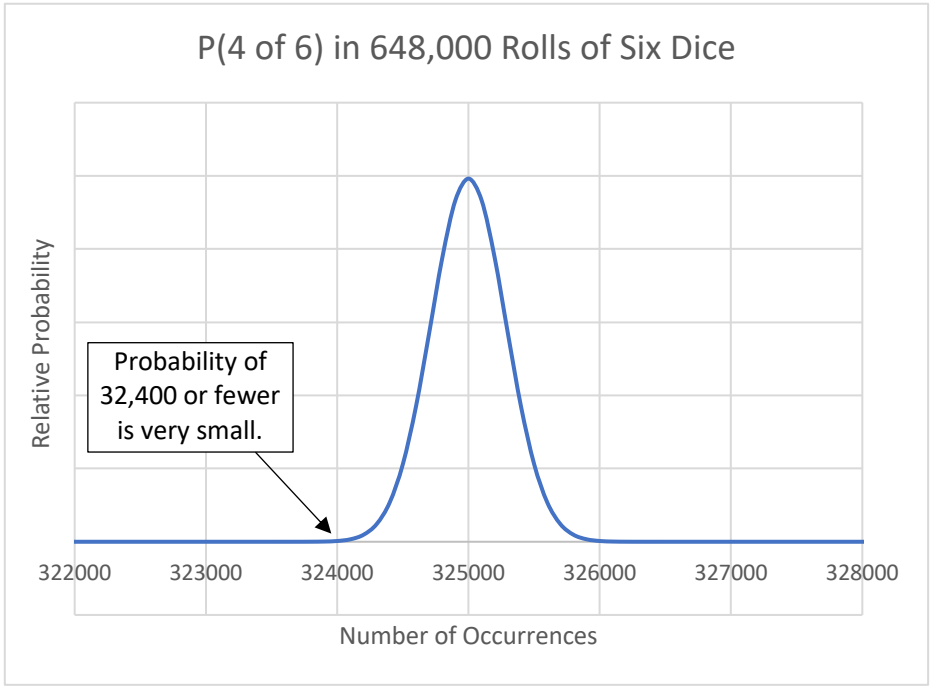
Let's try 648,000 rolls. If six dice are rolled 648,000 times, we would expect that four of the six numbers would appear in 325,000 of them.

If the probability of getting four in six numbers was actually 50%, we would expect four of the six numbers would appear in 324,000 of the 648,000 rolls.

The probability distribution is shown in the graph at the bottom of the page.

It can be seen that the probability of getting 324,000 or fewer is nearly zero.

These plots allow the problem of the gambler to be fully appreciated, and it demonstrates the extreme number of experiments needed to determine the probability of an event with significant precision.



The Real Tragedy of Lowering Educational Standards

By Ed Meyer

I teach physics at a University. We have an engineering program. All engineering majors are required to take the standard set of physics courses - one involving mechanics and thermosynamics and the involving electromagnetism and optics.

The level of preparedness of the incoming engineering majors has been dropping steadily over the past five years. Perhaps COVID contributed to this, perhaps it comes from the lowering of standards in high schools.

Recently 1/3 of the engineers flunked my freshman physics class. Many of the students that flunked simply did not work hard. Others were woefully underprepared and lacked basic mathematics and problem-solving skills.

As a result, an administrator told me that my exams are too hard for the students and if that many students flunk the next time I teach it, I will not be allowed to teach it anymore.

I was told that we have to meet the students at the level they are at. I'm fine with meeting them where they are at, but I would like to get them moving toward the level they need to be as quickly as possible.

I was told that "we don't want the students to feel stupid." I don't want them to feel stupid either, but I want them to understand that they have a long way to go to be an engineer and that they have to get to work.

I was told that "we don't want the students to be frustrated by problems that they have to struggle with." Frustration is a natural, evolved response of humans to challenging problems. This response evolved in rder for us to survive because when you think hard (System II thinking) you lose awareness of what is going on around you and this was dangerous during our long evolution. It today's society, the natural frustration response is vestigial. Fortunately, we have a mammalian brain that can overcome the natural frustration response with effort, determination and practice. The only way to develop the ability to struggle with challenging problems is to practice struggling with challenging problems.

Giving the students easy problems and leading them by the hand through them is not going to produce mentally tough engineers. Doesn't that make sense?

I think it is important to be honest with the students that are not doing well. We must make it clear to them that they are not on the path to success and that they are going to have to realign their priorities.

Some of my best success stories are from students that flunked my freshman physics class. They got the message and rededicated themselves to their mental development and went on to be very successful.

The real tragedy occurs when the curriculum is simplified to accommodate students that are worried about getting frustrated and feeling anxious and good students invest four years of their lives and about $120,000 to become an engineer. They have a 3.8 grade point average and they have glowing letters of receommendation. However, no engineering firm will hire them because the University's priority is not to produce gritty, tenacious, hard-working problem solvers, the university's priority is to not stress the students with challenging problems that give them anxiety and produce frustration.

It is clear that employers are very disappointed with recent college graduates. A recent article in Forbes was titled, "Bosses are firing Gen Z grads just months after hiring them—here's what they say needs to change."

If you know someone that is going to a university for specific job training, for example nursing, engineering, or acting, find out what percentage of their graduates are successfully working in their field of choice immediately after graduation.

If anyone representing the university says they don't have that information, that is a huge red flag. Either they don't have it (which they should), or they have it and they don't want to admit it.

Quote Acrostic

edmeyer.phd

Instructions: Fill in the words at the bottom from the clues. Then write those letters in the grid at the top to reveal a quote. Black squares indicate the end of a word and punctuation has been removed. When you're done the first letters of the answers to the clues will be the author of the quote.

QUOTE

1C	2I	3G	4L	5N	6E	7H	8J	9K	10A	■	11I	12G	13L	14H	15D	16M	17N	18K	19B	20F	21L
■	22H	23C	24H	25D	■	26J	27B	28L	29M	30F	■	31D	32B	33J	34I	35E	36J	37A	■	38K	39A
;	40F	41H	42G	43M	44K	45G	46J	47E	48C	■	49E	50F	51G	52A	53L	54J	■	55B	56I	57C	58M
■	59F	60M	61H	62L	63E	■	64K	65A	66M	■	67G	68C	69J	70D	71H	■	72C	73B	74E	75I	■

CLUES

Author→

A. Metal test — 52 10 39 65 37

B. Wood turner — 32 19 55 27 73

C. Insults — 72 57 48 23 68 1

D. Cleveland to Boston — 70 31 15 25

E. Tool with a pawl — 47 35 49 63 74 6

F. Hurled — 20 30 50 40 59

G. Motor — 45 67 42 3 12 51

H. Moment of _____ — 61 41 14 71 22 7 24

I. Specialized — 75 11 2 56 34

J. Avoid a road hazard — 8 26 46 33 69 54

K. Suffix for no, any, and some — 9 64 38 44 18

L. Kicked out — 4 13 28 62 53 21 36

M. Compulsive desire — 43 16 29 60

N. U.S. info security agency — 5 17 66 58