



# Making Progress on Grand Challenge TWO – Part IV

By Ed Meyer

In the last three issues we explored the solution space for Grand Challenge TWO, which is

*Six standard dice are rolled. What is the probability that exactly four of the possible numbers appear? Note that a gambler in the 1700s knew that the result was very close to 50%, but he nor any mathematician he asked could calculate it. Can you?*

In the previous issue we started to consider the rolls that contained two doublets and two singlets. The example we used was

Green	Blue	Red	Yellow	Orange	Purple

Let's focus on the position of the two doublets, and we'll do this one at a time. There are fifteen different positions for the two ones and they are listed below.

1-1-x-x-x-x	x-1-1-x-x-x	x-x-1-x-1-x
1-x-1-x-x-x	x-1-x-1-x-x	x-x-1-x-x-1
1-x-x-1-x-x	x-1-x-x-1-x	x-x-x-1-1-x
1-x-x-x-1-x	x-1-x-x-x-1	x-x-x-1-x-1
1-x-x-x-x-1	x-x-1-1-x-x	x-x-x-x-1-1

Notice how then two ones are carefully and methodically shuttled from the left to the right.

With the two ones in place, there are four spots for the two twos. Let's perform the same shuttling operation with two twos in four spaces.

2-2-x-x	x-2-2-x
2-x-2-x	x-2-x-2
2-x-x-2	x-x-2-2

There are six. The two remaining spaces are for the two singlets. There are six combinations for the two singlets.

3-4	3-5	3-6
4-5	4-6	5-6

Each of these has two permutations.

So, for every single position of the two doublets, there are twelve different permutations of the two singlets. Let's list them all for the sequences that start 1-1-2-2.

1-1-2-2-3-4	1-1-2-2-4-3
1-1-2-2-3-5	1-1-2-2-5-3
1-1-2-2-3-6	1-1-2-2-6-3
1-1-2-2-4-5	1-1-2-2-5-4
1-1-2-2-4-6	1-1-2-2-6-4
1-1-2-2-5-6	1-1-2-2-6-5

Each of these twelve permutations appears exactly once in the table of 46,656 possible rolls.

We have seen that there are 15 positions for the first doublet and six positions for the second doublet. This is 90 total positions of the two doubles and each of these has twelve possible singlet permutations. This means that there are

$$15 \times 6 \times 12 = 1080$$

cells in the table of 46,656 that contains a doublet of ones, a doublet of twos and two singlets.

Now we have to include the fact that here are a number of different doublets. They are

1-2	2-3	3-5
1-3	2-4	3-6
1-4	2-5	4-5
1-5	3-6	4-6
1-6	3-4	5-6

There are 15 and each of these has 1080 permutations. This is a total of

$$N_{2,2,1,1} = 15 \times 1080 = 16,200$$

If you are not 100% confident in this, you should invest some time working it out.





Now we can perform the final calculation. We have already counted 7200 permutations of the triplet and three singlets. Adding the 16,200 permutations of two doublets and two singlets, we get that 23,400 of the 46,656 possible rolls contain exactly four of the six possible numbers. This is

$$P = \frac{23,400}{46,646} = \frac{325}{648}$$

Which is very close to 50%.

Not surprisingly, there are many paths to this solution. Expert mathematicians will make short work of this problem by using equations with factorials that they have mastered.

It is interesting to note that many teachers of probability believe that young students must be trained to use the equation of probability if they are going to solve this challenging problem.

This is incorrect. Twelve-year-olds can solve this problem without relying on any standard equations. They just need to be able to count, they need mental stamina, and they have to be careful and methodical.

It may take weeks. That's the point. It is just like getting into physical shape. There are no shortcuts. There must be struggle. There is no progress without struggle.

Often, professional mathematicians will lament at the inefficiency of the method that was used to solve the problem. The point, however, is not to get the answer as quickly as possible. The point is to use the problem as a playing field on which to develop mental strength, mental stamina, and the ability to aggressively, efficiently, and confidently attack challenging problems.

In the next issue, we'll present another way to solve this problem.

**Test your knowledge of this version of the solution.**

1. Three dice are rolled. Each die has six possible outcomes, which means that there are 216 possible permutations. The lowest is 1-1-1 and the highest is 6-6-6.

a) What is the probability that all three numbers are different?

b) What is the probability that there is a pair and a singlet?

2. Four dice are rolled. Each die has six possible outcomes, which means that there are 1296 possible permutations. The lowest is 1-1-1-1 and the highest is 6-6-6-6.

a) What is the probability that all four numbers are different?

b) What is the probability that there is a pair and two singlets?

3. Six dice are rolled. Each die has six possible outcomes, which means that there are 46,656 possible permutations. The lowest is 1-1-1-1-1-1 and the highest is 6-6-6-6-6-6. How many of the 46,656 possible permutations contain three 1's, a 2, a 3 and a 4? Here is one possible permutation, but there are a lot more!

Green	Blue	Red	Yellow	Orange	Purple

4. Six dice are rolled. Each die has six possible outcomes, which means that there are 46,656 possible permutations. The lowest is 1-1-1-1-1-1 and the highest is 6-6-6-6-6-6. How many of the 46,656 possible permutations contain two 1's, two 2's, a 3 and a 4? Here is one possible permutation, but there are a lot more!

Green	Blue	Red	Yellow	Orange	Purple





# Quote Acrostic

[edmeyer.phd](http://edmeyer.phd)

**Instructions:** Fill in the words at the bottom from the clues. Then write those letters in the grid at the top to reveal a quote. Black squares indicate the end of a word and punctuation has been removed. When you're done the first letters of the answers to the clues will be the author of the quote.

## QUOTE

1G	2H	3E	4C	5B	6D	7G	8I	9G	10A		11C	12E	
13A	14C	15I	16D		17F	18F	19B	20H	21A	22E	23G	24D	25C
26I	27G	28E	29H	30C	31D	32D	33F	34D	35B		36C	37B	
	38H	39A	40E		41A	42F	43G	44F	45E		46I	47H	48C
49H	50E	51G	52H	53D	54A								

## CLUES

A. Original Inhabitant

10      41      39      21      13      54

B. Uncertain

5      19      37      35

C. Photographic

14      36      4      48      25      11      30

D. Mellifluous

24      16      53      31      6      32      34

E. Arrestee

12      50      40      28      3      22      45

F. Construct

42      44      18      33      17

G. Frontier settlement

9      23      7      1      43      27      51

H. Quicker to take offense

47      52      49      20      38      29      2

I. Ceremony

15      8      46      26

